# SIFAT-SIFAT TRANSFORMASI LAPLACE

a. Linearity

Fungsi waktu : C f(t)→C;konstanta

$$\mathcal{L}\, C\, f(t) = C\, F(s)$$

b. Superposisi

A dan b ; konstanta

$$\mathcal{L}\, f_1(t) = F_1(s)$$

$$\mathcal{L}\, f_2(t) = F_2(s)$$

$$\mathcal{L}\left[a\, f_1(t) + b\, f_2(t)\right] = a\, F_1(S) + b\, F_2(S)$$

c. Time Scaling

$$\mathcal{L}\, f\left(\frac{t}{a}\right) = a\, F(s)$$

d. Differensiasi

1. Di bidang t

   Fungsi waktu f(t)

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] = s\, F(s) - f(o)$$

$$\text{Bukti : } \mathcal{L}\left[\frac{d}{dt} f(t)\right] = \int_0^{\infty} \frac{d}{dt} f(t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} e^{-st}\, d\, f(t) \longrightarrow \text{dengan integrasi parsiil}$$

$$= e^{-st} f(t) \Big]_0^{\infty} - \int_0^{\infty} f(t)\, (-S)\, e^{-st}\, dt$$

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] = -f(o) + s\int_0^\infty f(t)\ e^{-st}\ dt$$

$$= s\,F(s) - f(o)$$

2. Di bidang s

$$F(s) = \mathcal{L}\ f(t)$$

$$\boxed{\frac{d}{dt} F(s) = -\mathcal{L}\left[t\ f(t)\right]}$$

Bukti :

$$\frac{d}{ds} F(s) = \frac{d}{ds}\int_0^\infty f(t)\ e^{-st}\ dt$$

$$= \int_0^\infty f(t)\ \frac{d}{ds}\ e^{-st}\ dt$$

$$= \int_0^\infty f(t)\ (-t)\ e^{-st} dt.$$

$$= -\int_0^\infty t\ f(t)\ e^{-st} dt.$$

$$= -\mathcal{L}\left[t\ f(t)\right]$$

e. Integrasi

Di bidanq t:→F(s)=£ t $_{(t)}$

$$\boxed{\mathcal{L}\left[\int f(t)\ dt\right] = \frac{F(s)}{s} - \frac{f(o)}{s}\quad dt}$$

Bukti :

$$F(s) = \int_0^\infty f(t)\ e^{-st}\ dt.$$

Berdasarkan integrasi Parsial :

$$\int_0^\infty u\, dv = u\, v \Big]_0^\infty - \int_0^\infty v\, dv$$

$$u = e^{-st} \qquad ; \quad v = s\, f(t)\, dt$$

$$du = -s\, e^{-st}\, dt \qquad ; \quad dv = f(t)\, dt$$

$$F(s) = e^{-st} \int f(t)\, dt \Big]_0^\infty - \int_0^\infty \int f(t)\, dt \; ( -s\, e^{-st}\, dt)$$

$$F(s) = - \int f(o)\, dt + s \int_0^\infty \int f(t)\, dt\; e^{-st}\, dt)$$

$$= - \int f(o)\, dt + s\, \mathcal{L} \left[ \int f(t)\, dt \right]$$

$$\boxed{\mathcal{L} \left[ s \quad f(t)\, dt \right] = \frac{F(s)}{s} - \frac{f(o)}{s}\, dt}$$

Carilah untuk bidang s

Tambahan fungsi differesiasi (bagian d):

$$\boxed{\mathcal{L} \left[ \frac{d}{dt} f(t) \right] = s\, F(s) - f(o)}$$

Dengan cara yang sama dapat dibuktikan:

$$\boxed{\mathcal{L} \left[ \frac{d^2}{dt^2} f(t) \right] = s^2\, F(s) - s\, f(o) - \frac{d}{dt} f(o)}$$

Secara Umum:

$$\mathcal{L}\left[\frac{d^n}{dt^n} f(t)\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) \ldots\ldots \frac{d^{n-1}}{dt^{n-1}} f(0)$$

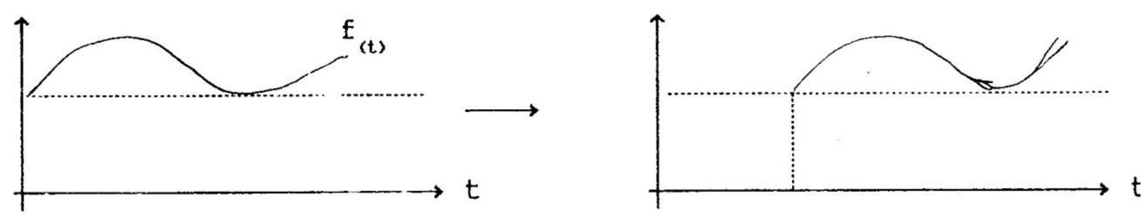
f. Translasi :

1. Dibidang t : $f(t) = f(t).\mu(t)$

   $f(t-\alpha) = f(t-\alpha).\mu(t-\alpha)$

   $\mu(t)$ = unit step function

$$\mathcal{L}\left[ f(t-a) \right] = \mathcal{L}\left[ f(t-a) \,.\, \mu(t-a) \right] = e^{-as} \; F(s)$$



Bukti :

$$F(s) = \int_0^\infty f(t)\, e^{-st}\, dt.$$

$$= \int_0^\infty f(\sigma)\, e^{-s\sigma}\, dt \longrightarrow \sigma = t-a$$

$$= \int_0^\infty f(t-a)\, e^{-s(t-a)}\, dt.$$

$$= e^{as} \int_0^\infty f(t-a)\, \mu(t-a)\, e^{-st}\, dt.$$

$$\text{Jadi} \quad \mathcal{L}\left[ f(t-a) \,.\, \mu(t-a) \right] = e^{-as}\, F(s)$$

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