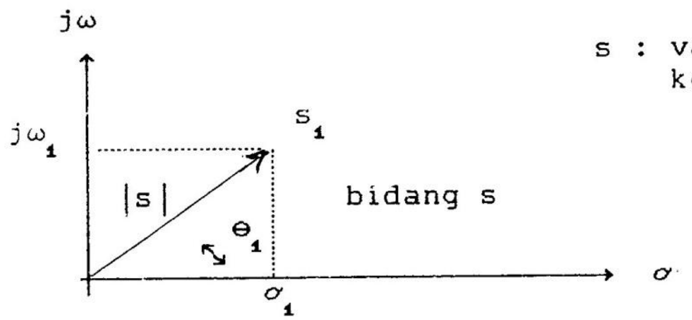
TRANSFORMASI LAPLACE

KLASIK		LAPLACE
Rangkaian / sistem		
↓		
Pers. Diferensial	⟶	Trasformasi Laplace
↓		↓
Klasik		$F(s) = \mathcal{L} f(t)$
↓		↓
Solusi (mis. $i(t)$)		Inverse-kan → $\mathcal{L}^{-1} f(t)$
		↓
		Solusi (mis. $i(t)$)

Jika kita mempunyai suatu fungsi $f_{(t)}$, kita kalikan dengan e^{-st}, kemudian di integrir terhadap waktu dengan batas-batas dari 0 sampai tak terhingga, maka diperoleh;

$$\mathcal{L} f(t) = F(s) = \int_0^\infty f(t)\, e^{-st}\, dt$$



s : variabel baru (bilangan komplek ($\sigma + j\omega$)

contoh berapakah F(s) dari fungsi-fungsi berikut ini

1. Diket : $f_{(t)} = \sin \omega t$.

$$F_{(s)} = \int_0^\infty \sin \omega t \; e^{-st} \, dt.$$

$$\sin \omega t = \frac{e^{+j\omega t} - e^{-j\omega t}}{2j}$$

$$= \frac{1}{2j} \left(e^{+j\omega t} - e^{-j\omega t} \right)$$

$$= \int^\infty \frac{1}{2j} \left(e^{+j\omega t} - e^{-j\omega t} \right) e^{-st} \, dt.$$

$$= \int_0^\infty \frac{1}{2j} \left(e^{(j\omega - s)t} - e^{-(j\omega + s)t} \right) dt$$

$$= \frac{1}{2j} \int_0^\infty e^{(j\omega - s)t} dt - \frac{1}{2j} \int_0^\infty e^{-(j\omega + s)t} dt$$

$$= \frac{1}{2j} \left[\frac{e^{(j\omega - s)t}}{(j\omega - s)} - \frac{e^{-(j\omega + s)t}}{-(j\omega + s)} \right]_0^\infty$$

$$F_{(s)} = \frac{1}{2j} \left[\frac{-1}{(j\omega - s)} + \frac{-1}{(j\omega + s)} \right]$$

$$= \frac{1}{2j} \left[\frac{-j\omega - s - j\omega + s}{(j\omega)^2 + s^2} \right]$$

$$= \frac{1}{2j} \left[\frac{+2j\omega}{(\omega)^2 + s^2} \right]$$

$$= \frac{\omega}{\omega^2 + s^2}$$

$$\boxed{\mathscr{L} \sin \omega t = \frac{\omega}{\omega^2 + s^2}}$$

b. Diket : $f(t) = \cos \omega t$

Jawab :
$$\cos \omega t = \frac{e^{+j\omega t} + e^{-j\omega t}}{2}$$

$$F(s) = \frac{1}{2} \int_{0}^{\infty} \left[e^{(j\omega - s)t} + e^{-(j\omega + s)t} \right] dt$$

$$= \frac{1}{2} \left[\frac{e^{(j\omega - s)t}}{(j\omega - s)} + \frac{e^{-(j\omega + s)t}}{-(j\omega + s)} \right]_{0}^{\infty}$$

$$= \frac{1}{2} \left[\frac{-1}{(j\omega - s)} + \frac{-1}{(j\omega + s)} \right]$$

$$= \frac{1}{2} \left[\frac{-j\omega - s + j\omega - s}{(j\omega)^2 + s^2} \right]$$

$$= \frac{1}{2} \left[\frac{-2s}{(j\omega)^2 + s^2} \right]$$

$$\boxed{\mathcal{L} \cos \omega t = \frac{s}{\omega^2 + s^2}}$$

c. Diket . $f(t) = e^{at}$

Jawab

$$F(s) = \int_0^\infty e^{at} \cdot e^{-st} \, dt.$$

$$= \int_0^\infty e^{(a-s)t} \, dt.$$

$$= \left. \frac{e^{(a-s)t}}{(a-s)t} \right]_0^\infty$$

$$= \frac{-1}{a-s} = \frac{1}{s-a}$$

$$\boxed{\mathcal{L}\, e^{at} = \frac{1}{s-a}}$$

a. Diket : f(t) = μ (t) unit step funftion

μ (t) = 0 → t < 0

μ (t) = 1→ t > 0

$$\mathcal{L}\, \mu(t) = \int_0 \mu(t) \cdot e^{-st} \, dt.$$

$$= \int_0^\infty 1\, e^{-st} \, dt.$$

$$= \left. \frac{1}{-s} e^{-st} \right]_0^\infty$$

$$\boxed{\mathcal{L}\, \mu(t) = \frac{1}{s}}$$

b. Diket : f(t) = V µ (t)

Jawab : £ V µ (t) = F (s) = $\frac{V}{s}$

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