Epidemiology of Lymphatic Filariasis Model in Kelurahan Jati Sampurna

Mathematical Model Transmission Filariasis Without Treatment

To simplify mathematical model, we have some assumption :

1. Virgin population.
2. The environment is small, it is village.
3. One species of worm.
4. One species of mosquito.
5. Hospes reservoir is ignored.
6. The environment factor is ignored.
7. Total population of human is constant.
8. Total population of mosquito is constant.
9. Every human and mosquito born healthy.

After transmission process, human population is divided in three subpopulation. First, subceptible human (S_h), the carier population (A), the chronic population (K). All human subpopulation is defined (N_h). The mosquitoes is divided in two subpopulation, first, subceptible mosquitoes (S_v) and infected mosquitoes (I_v). Both of mosquitoes population is defined (N_v).

Some factors in modelling construction are :

1. Recruitment rate human per second (R_h).
2. Natural death of human per second (μ_h).
3. Successful rate transmission from mosquitoes to susceptible human.
4. Biting rate on human caused by a mosquito per second (b).
5. The velocity of the appearance of symptoms per second (δ).
6. Recruitment rate mosquitoes per second (R_v).
7. Natural death of mosquito per second (μ_v).
8. Successful rate filaria transmission from human to susceptible mosquito (p_v).

Mathematical model without treatment are :

$$\frac{dS_h}{dt} = R_h - bI_v \frac{S_h}{N_h} p_h - \mu_h S_h$$

$$\frac{dA}{dt} = bI_v \frac{S_h}{N_h} p_h - \delta A - \mu_h A$$

$$\frac{dK}{dt} = \delta A - \mu_h K$$

$$\frac{dS_v}{dt} = R_v - bS_v \frac{A}{N_h} p_v - \mu_v S_v$$
$$\frac{dI_v}{dt} = bS_v \frac{A}{N_h} p_v - \mu_v I_v$$

Total Population of Human

Total population of human is $N_h = S_h + A + K$. The differensial equation of total population of human is $\frac{dN_h}{dt} = R_h - \mu_h N_h$. Base on assumption, we know $\frac{dN_h}{dt} = 0$ which is obtained $N_h = \frac{R_h}{\mu_h}$.

Total Population of Mosquitoes

Total population of mosquitoes are $N_v = S_v + I_v$. The differensial equation of total population of mosquitoes are $\frac{dN_v}{dt} = R_v - \mu_v N_v$. Base on assumption total population of human and mosquitoes are constant, then we have dynamical system

$$\frac{dA}{dt} = bI_v \frac{\frac{R_h}{\mu_h} - A - K}{\frac{R_h}{\mu_h}} p_h - \delta A - \mu_h A$$
$$\frac{dK}{dt} = \delta A - \mu_h K$$
$$\frac{dI_v}{dt} = b\left(\frac{R_v}{\mu_v} - I_v\right)\frac{A}{\frac{R_h}{\mu_h}} p_v - \mu_v I_v$$

We have non endemik stationer $E_1 = \left(A', K', I_v'\right) = (0,0,0)$ and endemic stationer $E_2 = \left(A^*, K^*, I_v^*\right)$ where

$$A^* = \frac{R_h\left(p_v b^2 \mu_h R_v p_h - \mu_v{}^2 R_h \delta - \mu_v{}^2 R_h \mu_h\right)}{p_v b \mu_h \left(\mu_v R_h \delta + bR_v \delta p_h + \mu_v R_h \mu_h + b\mu_h R_v p_h\right)}$$
$$K^* = \frac{\delta R_h\left(p_v b^2 \mu_h R_v p_v - \mu_v^2 R_h \delta - \mu_v^2 R_h \mu_h\right)}{p_v b \mu_h^2 \left(\mu_v R_h \delta + bR_v \delta p_h + \mu_v R_h \mu_h + b\mu_h R_v p_h\right)}$$
$$I_v^* = \frac{p_v b^2 \mu_h R_v p_h - \mu_v^2 R_h \delta - \mu_v^2 R_h \mu_h}{b\mu_v p_h\left(p_v b \mu_h + \mu_h \mu_v + \mu_v \delta\right)}$$

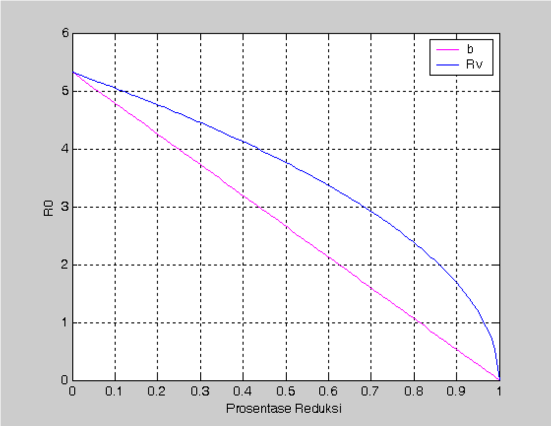
Existence of endemic stationer if

$$\frac{p_v b^2 \mu_h R_v p_h}{\mu_v^2 R_h (\delta+\mu_h)} > 1$$

The Basic Reproduction Number is

$$R_0 = \frac{b\sqrt{R_h(\delta+\mu_h) p_v \mu_h R_v p_h}}{R_h(\delta+\mu_h)\mu_v}$$

If the prosentation of R_0 is decreasing we have grafic



Filariasis Transmission Mathematical Model With Treatment

This model use assumption

1. Assumption without treatment still used.
2. Survei to find chronic is done if find one chronic then implemented screening to whom who live around the chronic one.
3. If among n person who implemented mass drug administration.
4. The chronic people is negatif microfilaria.

Mathematical model with treatment is generalizing from mathematical model without treatment with adding treatment process. Mathematical filaria transmission model is

$$\frac{dS_h}{dt} = R_h + \alpha\frac{A}{N_h}nK - bI_v\frac{S_h}{N_h}p_h - \mu_h S_h$$

$$\frac{dA}{dt} = bI_v\frac{S_h}{N_h}p_h - \delta A - \alpha\frac{A}{N_h}nK - \mu_h A$$

$$\frac{dK}{dt} = \delta A - \mu_h K$$

$$\frac{dS_h}{dt} = R_v - bS_v\frac{A}{N_h}p_v - \mu_v S_v$$

$$\frac{dI_v}{dt} = bS_v\frac{A}{N_h}p_v - \mu_v I_v$$

Base on total population of human and mosquitoes is constant so we have filaria transmission dynamical system

$$\frac{dA}{dt} = bI_v \frac{\frac{R_h}{\mu_h} - A - K}{\frac{R_h}{\mu_h}} p_h - \delta A - \alpha \frac{A}{\frac{R_h}{\mu_h}} nK - \mu_h A$$

$$\frac{dK}{dt} = \delta A - \mu_h K \qquad (1)$$

$$\frac{dI_v}{dt} = b\left(\frac{R_v}{\mu_v} - I_v\right)\frac{A}{\frac{R_h}{\mu_h}} p_v - \mu_v I_v$$

From the equation above we have non endemic stationer $T_1 = \left(A^{'}, K^{'}, I_v^{'}\right) = (0,0,0)$ and the condition of existence endemic stationer is

$$b^2 \frac{R_v}{R_h} \frac{\mu_h}{(\delta + \mu_h)\mu_v^2} p_v p_h > 1$$

Basic reproduction number is

$$R_0 = b\frac{\sqrt{R_h(\delta + \mu_h) p_v \mu_h R_v p_h}}{R_h(\delta + \mu_h)\mu_v} \qquad (2)$$