Jurnal Pengajaran MIPA
Vol. 21, No. 2, Oktober 2016, hlm. 109-114
ISSN 1412-0917 (print)/ 2443-3616 (online)

DOI: 10.18269/jpmipa.v21i2.817

BETWEEN FORMAL AND INFORMAL THINKING: THE USE OF ALGEBRA FOR SOLVING GEOMETRY PROBLEMS FROM THE PERSPECTIVE OF VAN HIELE THEORY

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ABSTRACT

This study investigated primary education master program students' problem solving strategies and their formal and informal thinking ability when dealing with geometry problems that require the use of algebra in its solution processes. In order to do so, an explorative study through individual written test, observation, and field notes, involving 47 primary education master program students was carried out. The perspective of Van Hiele theory on the development of geometric thought was used to interpret student formal and informal thinking strategy when dealing with geometry problems. The results showed that more than half of the students used informal rather than formal algebraic strategies in solving geometry problems; when students used algebraic strategies, their work were imperfect as they still made mistakes in applying the strategies. In the light of Van Hiele theory, it can be concluded that students' level of thinking are still in between formal and informal thinking when dealing with geometry problems.

Keywords: algebra; geometry; formal and informal thinking;Van Hiele theory

ABSTRAK

Penelitian ini menyelidiki strategi pemecahan masalah mahasiswa program magister pendidikan dasar serta kemampuan berpikir formal dan informal mereka ketika menyelesaikan soal geometri yang memerlukan penggunaan aljabar dalam proses penyelesaiannya. Untuk mencapai tujuan ini, studi eksploratif melalui tes individu tertulis, observasi dan catatan lapangan dilakukan dengan melibatkan 47 mahasiswa program magister pendidikan dasar. Teori Van Hiele digunakan untuk menginterpretasi kemampuan berpikir formal dan informal mahasiswa dalam menyelesaikan soal-soal geometri. Hasil penelitian menunjukkan bahwa lebih dari separuh mahasiswa menggunakan strategi-strategi informal ketimbang strategi-strategi aljabar formal dalam proses penyelesaian soal-soal geometri; ketika mahasiswa menggunakan strategi-strategi aljabar, proses penyelesaian yang mereka lakukan tidak sempurna, dan masih melakukan kekeliruan-kekeliruan dalam menerapkan strategi tersebut. Berdasarkan tinjauan teori Van Hiele, dapat disimpulkan bahwa kemampuan berpikir mahasiswa masih berada pada kemampuan antara formal dan informal ketika menyelesaikan soal-soal geometri.

Kata kunci: aljabar; geometri; berpikir formal dan informal; teori Van Hiele

How to cite: Jupri, A., & Syaodih, E. (2016). Between Formal and Informal Thinking: The Use of Algebra for Solving Geometry Problems from the Perspective of Van Hiele Theory. *Jurnal Pengajaran MIPA*, 21(2), 109-114.

INTRODUCTION

The importance of geometry has been reflected in primary schools mathematics curriculum (National Council of Teachers of Mathematics-NCTM, 2000; Indonesian Ministry of Education and Culture, 2013) as it considered as a rich area to foster students' reasoning and problem solving skills (Herskowitz, 1998; NCTM, 2000; Howse and Howse, 2015). Thus, teachers as an influencing factors in mathematics teaching and learning (Ball, Hill, and Bass 2005; Herbst, 2006; Ng, 2011) has to effectively exploited geometry role in developing students' reasoning and problem solving skills. Unfortunately, teachers still found difficulties in providing, solving, and delivering non-routine geometry problems which require higher order thinking and problem solving skills (Szetela and Nicol, 1992), lack in mathematical knowledge for teaching geometry (Eli, Mohr-Schroeder, and Lee, 2013) and some even still perceived geometry as difficult topic to teach (Barrantes and Blanco, 2006). Due to their pivotal role in mathematics teaching and learning, the contributed factors that will hinder or facilitate learning should be appropriately addressed (Herbst, 2006). Previous study found that teachers' knowledge contributed to their ability in teaching geometry (Ng, 2011) and our preliminary study also found that teachers themselves (in service or pre service teachers) still encounter difficulties in solving geometry problems. Measuring teachers' ability in solving geometry problems will only give us a glimpse of what they can or cannot do, yet not an understanding of their thinking processes in solving geometry problems. Therefore, in order to gain a better insight on teachers' geometry ability as a whole, in this current study we investigated not only their ability in solving geometry problems but also *how* they solve geometry problems.

Two main theoretical frameworks were used, the theory of Van Hiele- an influential theory to evaluate students' geometric thought (Breyfogle and Lynch, 2010; Burger and Shaughnessy, 1986; Gutierrez, Howse and Howse, 1991; Teppo, 1991; Van Hiele, 1999; 1986), and algebraic strategies for solving geometry problems. Van Hiele theory classifies student geometric thought into five levels: visualization, analysis, abstraction, deduction, and rigor (Van Hiele, 1986; 1999). Level O (visualization) is a level where student describes basic geometric concepts by visual considerations of the concept as a whole without explicit view to properties of its components. For example, the student recognizes a rectangle by its form and considers it as a different shape from a square. In Level 1 (analysis), student describes basic geometric concepts by an informal analysis of component's parts and properties. For example, student recognizes that a rectangle has four sides and right angles, but its properties are not yet ordered. In contrary, in Level 2 (abstraction), student is already able to put in order properties of geometric concepts: one property follows another. For example, a square is recognized as a rectangle because the square has all properties of the rectangle, although, the intrinsic meaning of the deduction is not yet understood by the student. In other words, the student is not yet able to prove properties of geometric concepts deductively in a formal way. At Level 3 (deduction), student is able to reason deductively within the context of mathematical system, i.e., to think with undefined terminologies, axioms, definitions, and theorems. In other words, student is able to think in a formal manner. For example, student is able to prove that two diagonals of a rectangle have the same lengths. The last level or Level 4 (rigor) is a level where student is able to compare different geometry systems. For example, student is able to compare between Euclidean and non-Euclidean geometry without using concrete models. The five levels describe a progresssion of student geometric thinking from a concrete visual level to an increasingly sophisticated level of description, analysis, abstraction and proof (Van Hiele, 1986; 1999).

A fundamental role in the process of solving a mathematical problem is referred to mathematization (De Lange, 2006; Van den Heuvel-Panhuizen, 2003). If the problem is in the context of geometry and the mathematization process utilizes algebra, we call this process as an algebraic strategy for solving geometry problem. According to De Lange (2006), the mathematization process, in which a student uses algebra in the context of geometry, is a cyclical process which starts with a geometry problem within the geometry world. Next, student tries to identify relevant mathematics and reorganizes problem into an algebraic model within algebra world. Then the model is solved using algebraic rules and manipulations. Finally, the solution is reinterpreted into the initial geometry context. By using Van Hiele theory and algebraic strategies as theoreticcal frameworks, pre service and in service primary teachers' abilities in dealing with geometry problems, what types of thinking and strategies do they use when solving geometry problems can be thoroughly investigated.

METHODS

The study reported in this paper was a part of a larger qualitative study carried out through written test, observation, and field notes. The individual written test lasted for 60 minutes and involved 47 students enroll in primary education master program. Out of 47 students, 40 students are experienced primary school teachers (in service teachers) and the rest are prospective lecturers or teacher trainers (pre service teachers). The written test consisted of four geometry tasks about area and perimeter of triangles, squares, and rectangles. The tasks are adapted from mathematics Olympiad problems for primary school students (Sanjaya and Wijaya, 2007; Tampomas and Saputra, 2006).

The collected data from written test were students' written solutions, scrap papers, and field notes. These data were then analyzed through two steps. First, the student written test' solutions were grouped into units of analysis. One unit of analysis includes one task and its corresponding solution, and as there are four tasks and 47 students, there are a total of 188 units of analysis. The analysis includes determining whether students' answers are correct and identifying their problem solving strategies. Secondly, students' answers were then interpreted using Van Hiele theory.

RESULTS AND DISCUSSION

This section presents results of the data analysis which focus on two geometry problems inviting an explicit use of algebraic strategies in the solution processes. The two problems are *the square-and-rectangle* problem and *the triangle-and-rectangle* problem.

The square-and-rectangle problem

In this problem, students were asked to determine perimeter of rectangles and a square (Figure 1).

Q: *If a square is divided into three congruent rectangles and each rectangle has a perimeter of 160 cm, find the perimeter of the square!*

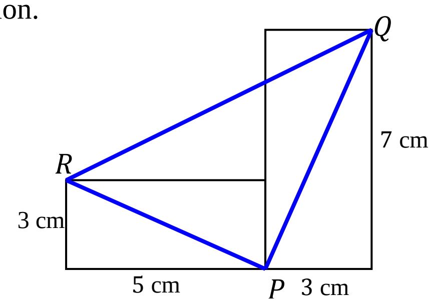
Figure 1. The square-and-rectangle problem

Out of 47 students participated in this study, 23 students solved the task correctly. Concerning solution strategies, 16 students used algebraic strategies, 29 students used informal guess-and-check strategies, and two students provided only answers. Figure 2a-b shows student written work using algebraic strategies in which Figure 2a shows an example of correct solution while Figure 2b shows an example of incorrect solution. From the perspective of Van Hiele theory, the use of algebraic strategies shows a formal deductive thinking, i.e the level 3 (e.g., Breyfogle and Lynch, 2010; Burger and Shaughnessy, 1986). This showed that students have reached the formal deductive thinking for the subtopic of the perimeter of the rectangle and the square (Breyfogle and Lynch, 2010).

Figure 3a-b shows student written work using guess-and-check strategies in which Figure 3a show an example of correct answers while Figure 3b show an example of incorrect answers. In the light of the Van Hiele theory, the use of guess-and-check strategies shows an informal thinking, i.e the level 2 (e.g., Breyfogle and Lynch, 2010; Burger and Shaughnessy, 1986). The use of informal strategy might be caused by students learning experience in which algebra is rarely use in solving mathematics problems; or because the task is still relatively easy to be guessed and checked. Another reason is that the task does not require students to use more formal, algebraic strategies. The use of informal strategies might also indicate that the student found difficulties in translating the problem into an algebraic model (Jupri and Drijvers, 2016).

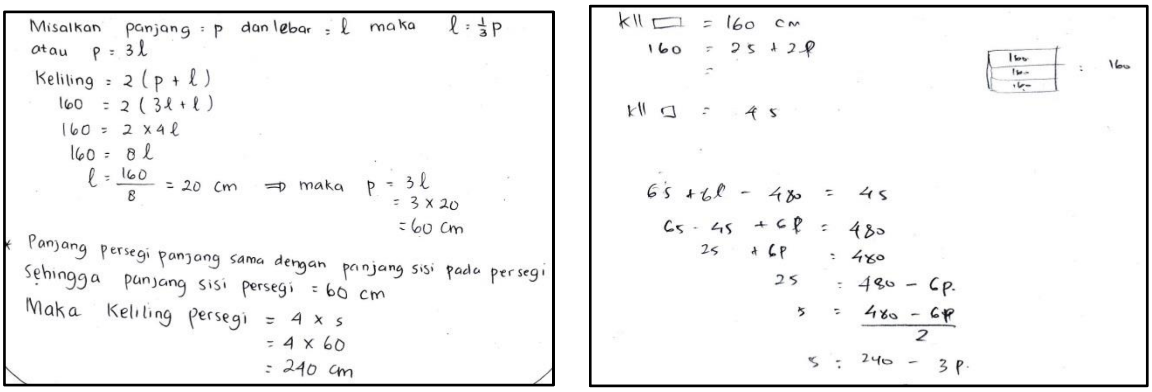
The triangle-and-rectangle problem.

The triangle-and-rectangle problem examines student problem solving skills about the area of a triangle and rectangles (Figure 2). Out of 47 students, only six students solved the task correctly. This indicates that the task is relatively difficult for most of the students. Furthermore, although 46 students used algebraic strategies to solve the task, 40 students did it incorrectly. The mistakes occurred, for instance, because students did not check whether the triangle PQR is a right triangle or not. In this case, students applied the Pythagorean theorem to the triangle PQR, which is not a right triangle, $PR^2 + PQ^2 \neq RQ^2$. Figure 4a shows an example of correct solution while Figure 4b shows an example of incorrect solution.



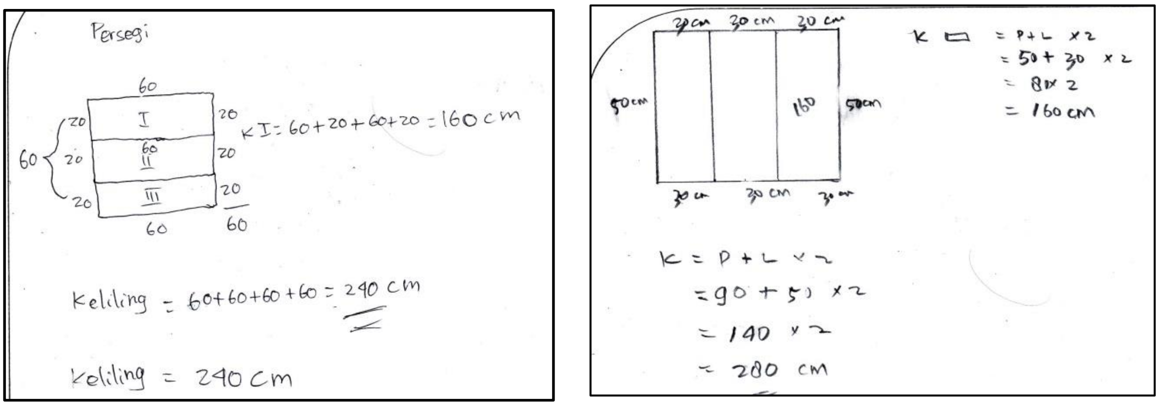
Q: Given the two rectangles above, find the area of triangle PQR!

Figure 2. The triangle-and-rectangle problem



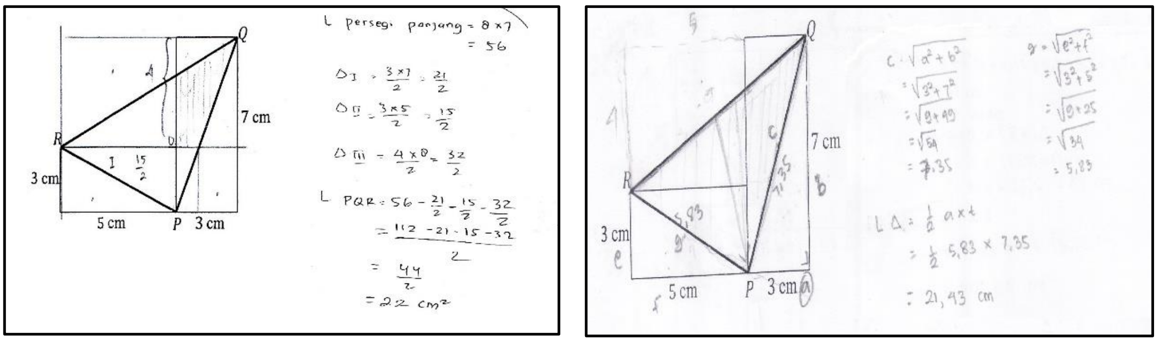
(2a) (2b)

Figure 2a-b. Student Works using Algebraic Strategies, Correct Answer (2a) and Wrong Answer (2b)



(3a) (3b)

Figure 3a-b. Student Works using Guess-and-check Strategies, Correct Answer (3a) and Wrong Answer (3b)



(4a) (4b)

Figure 4a-b. Student Works on the triangle-and-rectangle problem, Correct Answer (4a) and Wrong Answer (4b)

Taking the results into account, the use of algebraic strategies – as formal deductive thinking according to Van Hiele theory (Van Hiele, 1986, 1999) – is not yet perfect. The application of the Pythagorean theorem to a non-right triangle PQR, for instance, indicates that students do not understand when to use the theorem properly. In other words, students are still in the informal deductive thinking (level 2), and do not yet reach the formal deductive thinking (level 3). This condition occurs because the students have limited experience in applying formal algebraic strategies in problem solving processes. As a result, the deductive level of geometric thought (level 3) is not yet reached by the students (see Van Hiele, 1986, 1999).

CONCLUSION

The study reported in this article was carried out to investigate master students' problem solving strategies when dealing with geometry problems inviting the use of algebra in its solution processes, and to interpret their thinking from the perspective of Van Hiele theory. We found that more than half used informal guess-and-check strategies in solving geometry problems; and even when they use algebraic strategies they still made mistakes in carrying out the strategies, such as by applying the Pythagorean Theorem improperly. From the perspective of Van Hiele theory, the use of informal strategies indicates that the students have only reached the level 2 (abstraction), whereas the use of formal algebraic strategies suggests that the students have arrived at the level 3 (deduction). Based on the analysis, we conjecture that the participated students have only achieved between formal and informal thinking when dealing with geometry problems.

To improve the learning and teaching of geometry, we recommend the use of geometry problems that require explicit uses of algebra in its solution processes. In this way, we can expect the students to get accustomed with solving geometry problems which require the use of algebraic strategies either explicitly or implicitly. As a consequence, students' thinking will evolve from an informal thinking to a more formal one.

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