Twisted Toeplitz Algebras

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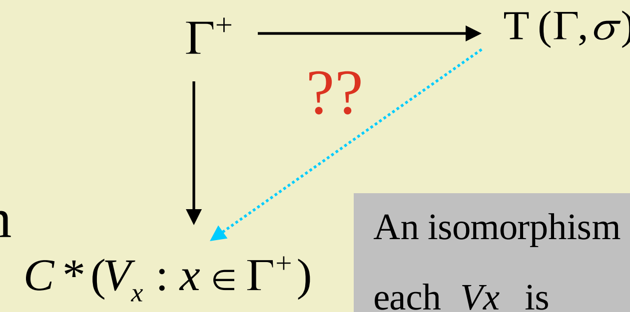
Given in a Seminar
At Chiba University
2007

The Universal property of Twisted Toeplitz Algebras

Corollary II.15

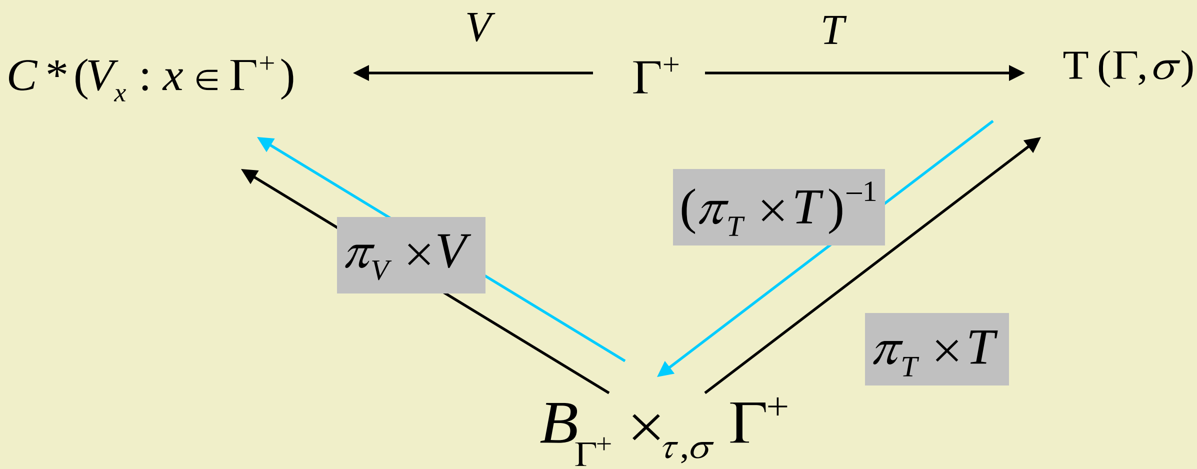
Γ: totally ordered abelian group, σ: cocycle on Γ.

$\mathrm{T}(\Gamma,\sigma)$ is universal for isometric-σ representation of Γ^+.



An isomorphism if each Vx is nonunitary

Proof :



The Universal property of Twisted Toeplitz Algebras

Corollary II.16

σ, ω are cocycles on Γ such that $[\sigma] = [\omega]$, then $\mathrm{T}(\Gamma, \sigma) \cong \mathrm{T}(\Gamma, \omega)$.

Proof :

$$\begin{array}{ccccc}
\mathrm{T}(\Gamma,\sigma) & \longleftarrow & \Gamma^{+} & \longrightarrow & \mathrm{T}(\Gamma,\omega) \\
\uparrow\downarrow & & & & \uparrow \\
B_{\Gamma^{+}} \times_{\tau,\sigma} \Gamma^{+} & & \longrightarrow & & B_{\Gamma^{+}} \times_{\tau,\varpi} \Gamma^{+}
\end{array}$$

Invariant Ideals

Theorem II.20

A: C*-alg, Γ: totally ordered abelian group, $\alpha: \Gamma^{+} \to \text{Endo}\, A$ action,

I : extendibly $\alpha -$invariant ideal of. A,

∴ there is an exact dp. barisan eksak pendek

$$0 \to I \times_{\alpha|_I, \sigma} \Gamma^{+} \to A \times_{\alpha, \sigma} \Gamma^{+} \to A / I \times_{\tilde{\alpha}, \sigma} \Gamma^{+} \to 0.$$

Proposition II.21

Terdapat barisan eksak pendek

$$0 \to B_{\Gamma^{+}, \infty} \times_{\tau|_{B_{\Gamma^{+}, \infty}}, \sigma} \Gamma^{+} \to B_{\Gamma^{+}} \times_{\tau, \sigma} \Gamma^{+} \to (B_{\Gamma^{+}} / B_{\Gamma^{+}, \infty}) \times_{\tilde{\tau}, \sigma} \Gamma^{+} \to 0.$$

Corollary II.22

Tdp. barisan eksak pendek

$$0 \to \mathrm{C}(\Gamma, \sigma) \to \mathrm{T}(\Gamma, \sigma) \to C^{*}(\Gamma, \sigma) \to 0.$$

Struktur dari $\mathcal{T}(\Gamma, \inf\sigma)$

Theorem III.1.

$I:$ ideal urutan dr Γ, $\sigma:$ kosikel pd Γ, $C(\Gamma, I, \inf\sigma)$ ideal dr $\mathcal{T}(\Gamma, \inf\sigma)$ dibangun $\left\{1\text{-}T_x T_x^* : x \in I^+\right\}$.

$\therefore$ Tdp. barisan eksak pendek

$$0 \to C(\Gamma, I, \inf\sigma) \to \mathcal{T}(\Gamma, \inf\sigma) \to \mathrm{Ind}_{I^\perp}^{\hat{\Gamma}}(\mathcal{T}(\Gamma/I, \sigma), \alpha^{\Gamma/I}) \to 0.$$

Sketsa Bukti

Menggunakan sifat universal alj. Toeplitz

$$V : \Gamma^+ \to C(\hat{\Gamma}, \mathcal{T}(\Gamma/I, \sigma)),\ V_s(\gamma) := \gamma(s) T_{q(s)}^{\Gamma/I} \text{ rep. isometri-}\sigma$$

$$\Downarrow$$

tdp. hom. $\phi_V : \mathcal{T}(\Gamma, \inf\sigma) \to C(\hat{\Gamma}, \mathcal{T}(\Gamma/I, \sigma))$, $\phi_V(T_s) = V_s$.

1. ϕ_V surjeksi pada $\mathrm{Ind}_{I^\perp}^{\hat{\Gamma}}(\mathcal{T}(\Gamma/I, \sigma))$: Lemma III.4
2. $\ker \phi_V = C(\Gamma, I, \inf\sigma)$

Adji (2000): Produk silang, ideal invarian

$$0 \to C_{I^+} \times_\tau \Gamma^+ \to B_{\Gamma^+} \times_\tau \Gamma^+ \to \left(B_{\Gamma^+} / C_{I^+}\right) \times_{\tilde{\tau}} \Gamma^+ \to 0.$$