STUDY ON SUPPRESSING FLOW-INDUCED VIBRATIONS OF TWO-MASS SYSTEM WITH PARAMETRIC EXCITATION BY USING AVERAGING METHOD

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Abstract. The possibility of suppressing self-excited vibrations of mechanical systems using parametric excitation is discussed. We consider a two-mass system of which the main mass is excited by a flow-induced, self excited force. A single mass which acts as a dynamic absorber is attached to the main mass and, by varying the stiffness between the main mass and the absorber mass, represents a parametric excitation. It turns out that for certain parameter ranges full vibration cancellation is possible. Using the averaging method the non-linear system is investigated producing as non-trivial solutions stable periodic solutions. In the case of a small absorber mass we have to carry out a second-order calculation.

1. Introduction

Suppressing flow-induced vibrations by using a conventional spring-mass absorber system has often been investigated and applied in practice. It is also well-known that self-excited vibrations can be suppressed by using different kinds of damping, see [Tondl (1991); Tondl, Kotek, Kratochvil(2001)]. However, only little attention has been paid to vibration suppression by using interaction of different types of excitation.

In this paper we solve a two-mass system formulated by Tondl, see [Ecker and Tondl (2000)]; the model is described in section 2. We will use the averaging method [Sanders and Verhulst (1985)]. The first order approximation is used to analyze the conditions for full vibration suppression. It turns out that full vibration cancellation is possible in an open parameter set. This is illustrated analytically. In sections 5 and 6 we study the stability and bifurcations of the trivial solution. Finally, in section 7 we return to the realistic problem of a small absorber mass. A second-order approximation has to be calculated in this case with as a result that, although full vibration cancellation is impossible, a fairly large part of vibration quenching can be achieved.

2. The Model

Consider a two-mass system consisting of a main mass m_2 which is in flow-induced vibration and an absorber mass m_1 which is attached to the main mass by a spring-damper element, see Figure 1. The elastic mounting $k(t)$ of the absorber mass is a combination of a spring and a device operating such that the stiffness $k(t)$ is changed periodically. Damping is represented by the linear viscous damper c_1. The main mass m_2 is supported by a spring with constant stiffness k_2; it has a linear viscous damper with damping parameter c_2. In actual constructions one usually has $m_1 < m_2$.

A flow-generated self-excited force is acting on the main mass m_2 with critical flow velocity U_c and a limited vibration amplitude in the over-critical region; as usual it is represented by a Rayleigh force.

The displacements of mass m_1 and mass m_2 are denoted by the coordinates y_1 and y_2, respectively. The variation of the stiffness of the absorber element is supposed to be a harmonic function with a small amplitude.

This system is represented by the following nonlinear equations of motion

$$\begin{aligned} &m_1 y_1'' + c_1(y_1' - y_2') + k_1(1 + \varepsilon \cos \omega\tau)(y_1 - y_2) = 0, \\ &m_2 y_2'' - c_1(y_1' - y_2') - k_1(1 + \varepsilon \cos \omega\tau)(y_1 - y_2) + c_2 y_2' + k_2 y_2 - b_\circ U^2(1 - \gamma_\circ {y_2'}^2) y_2' = 0. \end{aligned} \tag{2.1}$$

where ε is a small positive parameter, $0 < \varepsilon << 1$. In the decoupled system, where we only consider vibrations of the main mass m_2, self-excited vibrations occur if $c_2 - b_\circ U^2 < 0$.

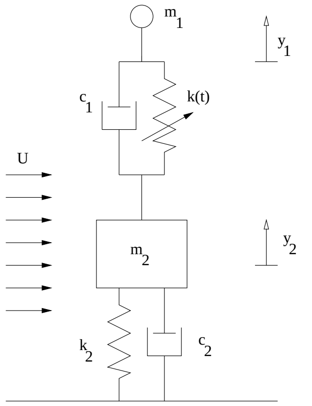


FIGURE 1. System consisting of a flow-excited main mass m_2 and a vibration absorber m_1 with time-dependent connecting stiffness $k(t)$.

3. TRANSFORMATION OF THE SYSTEM TO A STANDARD FORM

Dimensionless coordinates x_j can be defined with respect to a given reference value $y_\circ$: $x_j = \frac{y_j}{y_\circ}$ $j = 1, 2$. By introducing the characteristic parameters of the system $\bar{\omega}^2 = \frac{k_1}{m_1}$, ${\omega_\circ}^2 = \frac{k_2}{m_2}$, $\eta = \frac{\omega}{\omega_\circ}$, $Q^2 = \frac{\bar{\omega}^2}{{\omega_\circ}^2}$, and by using the time-transformation $\omega_\circ t = \tau$, the following dimensionless form of system (2.1) is obtained

$$\begin{aligned} &x_1'' + \kappa_1(x_1' - x_2') + Q^2(1 + \varepsilon \cos \eta\tau)(x_1 - x_2) = 0, \\ &x_2'' - M\kappa_1(x_1' - x_2') - MQ^2(1 + \varepsilon \cos \eta\tau)(x_1 - x_2) + \kappa_2 x_2' + x_2 - \beta V^2(1 - \gamma {x_2'}^2) x_2' = 0. \end{aligned} \tag{3.1}$$

where $\kappa_1 = \frac{c_1}{m_1 \omega_\circ}$, $\kappa_2 = \frac{c_2}{m_2 \omega_\circ}$, $\beta = \frac{b_\circ {U_\circ}^2}{m_2 \omega_\circ}$, $V^2 = \frac{U^2}{{U_\circ}^2}$, $\gamma = \gamma_\circ {\omega_\circ}^2$, $M = \frac{m_1}{m_2}$.

Parameter $U_\circ$ is a chosen reference value for the flow velocity. When $U_\circ$ reaches the critical flow velocity $U_c = \sqrt{c_2/b_\circ}$, the relative critical flow velocity is $V_c = 1$.

In order to transform the system into a standard form and to make the size of the parameters more explicit, we scale $\kappa_{1,2} = \varepsilon\bar{\kappa}_{1,2}$, and $\beta = \varepsilon\bar{\beta}$ while assuming that the other parameters are $O(1)$ with respect to ε. However, in quite a number of applications the absorber mass m_1 will be small with respect to the main mass m_2; we shall return to this case in section 9. If $\varepsilon = 0$, the linear parts of (3.1) now depend on the mass ratio M and the frequency ratio Q. Note, that if $\varepsilon > 0$, three frequencies play a part. Using the linear transformation

$x_1 = \bar{x}_1 + \bar{x}_2, \quad x_2 = a_1\bar{x}_1 + a_2\bar{x}_2$. leads to the standard form

$$\bar{x}_1'' + \Omega_1^2\bar{x}_1 = -\frac{\varepsilon}{a_1 - a_2}F_1(\bar{x}_1, \bar{x}_1', \bar{x}_2, \bar{x}_2', \eta\tau),$$
$$\bar{x}_2'' + \Omega_2^2\bar{x}_2 = -\frac{\varepsilon}{a_1 - a_2}F_2(\bar{x}_1, \bar{x}_1', \bar{x}_2, \bar{x}_2', \eta\tau), \tag{3.2}$$

where the natural frequencies of the linearized system without damping and for $\varepsilon = 0$, Ω_1, Ω_2 and $a_{1,2}$ are depending on M and Q. Note that the functions F_1 and F_2 are depending on the parametric excitation frequency η, and that the following conditions hold $\Omega_2 > \Omega_1, \quad a_1 a_2 = -M, \quad 0 < a_1 < 1, \quad$ and $\quad a_2 < -M$.

4. The Normal Form by Averaging

We will use the method of averaging to study the system near the *combination resonance* $\Omega_2 - \Omega_1 = \eta_\circ$. Transforming $t \rightarrow \eta\tau$ and allowing detuning near $\eta_\circ$ by putting $\eta = \eta_\circ + \varepsilon\bar{\sigma}$. system (3.2) becomes to first order in ε

$$\ddot{\bar{x}}_1 + \omega_1^2\bar{x}_1 = -\frac{\varepsilon}{(a_1 - a_2)\eta_\circ^2}\bar{F}_1(\mu, \bar{x}_1, \dot{\bar{x}}_1, \bar{x}_2, \dot{\bar{x}}_2, t),$$
$$\ddot{\bar{x}}_2 + \omega_2^2\bar{x}_2 = -\frac{\varepsilon}{(a_1 - a_2)\eta_\circ^2}\bar{F}_2(\mu, \bar{x}_1, \dot{\bar{x}}_1, \bar{x}_2, \dot{\bar{x}}_2, t). \tag{4.1}$$

where $\omega_{1,2} = \frac{\Omega_{1,2}}{\eta_\circ}$ and $\mu = (Q_{12}, Q_{21}, \theta_{11}, \theta_{22}, B)$. Q_{12}, Q_{21}, θ_{11}, θ_{22}, and B are depending on parameters. To study the behavior of the solutions, we transform $\bar{x}_1 = u_1\cos\omega_1 t + v_1\sin\omega_1 t, \quad \dot{\bar{x}}_1 = -\omega_1 u_1\sin\omega_1 t + \omega_1 v_1\cos\omega_1 t, \bar{x}_2 = u_2\cos\omega_2 t + v_2\sin\omega_2 t, \quad \dot{\bar{x}}_2 = -\omega_2 u_2\sin\omega_2 t + \omega_2 v_2\cos\omega_2 t$.

This transformation is useful when studying the stability of the trivial solution of the system; stability implies the possibility of vibration cancellation. After averaging over 2π and then rescaling time by a factor $\frac{\varepsilon}{2(a_1-a_2)\eta_\circ^2}$, we obtain the averaged normal form $\dot{U} = G(\mu, U)$ where $U = (u_1 \quad u_2 \quad u_3 \quad u_4)^T$ and $G = (G_1 \quad G_2 \quad G_3 \quad G_4)^T$.

5. Conditions for Vibration Cancellation:Linear Case

Systems involving interaction of self-excitation and parametric excitation have been studied in [Tondl (1978); (1991);(1997);(1998)].

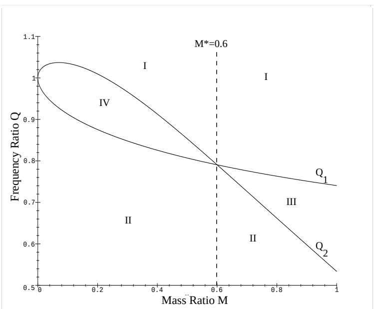


FIGURE 2. Boundaries of θ_{11} and θ_{22} in the (M, Q)-plane for $\varepsilon = \kappa_1 = \beta = 0.2$, $\kappa_2 = 0.1$, $V = \sqrt{2.1}$ and $\gamma = 4$. The curves Q_1 and Q_2 correspond with $\theta_{11} = 0$ and $\theta_{22} = 0$, respectively. Region I, $\theta_{11} < 0$ and $\theta_{22} > 0$. Region II, $\theta_{11} > 0$ and $\theta_{22} < 0$. Both θ_{11} and θ_{22} are positive in region III and they are negative in region IV. On the right side of the line $M^\star = 0.6$, $\theta_{11} + \theta_{22} > 0$ and $\theta_{11} + \theta_{22} < 0$ on the left side.

In the methods used there an implicit assumption on the magnitude of the parameters corresponds with our assumptions in the preceding section; in section 8 this will change. Here we present an independent analysis of the stability of the trivial solution based on the averaged normal.

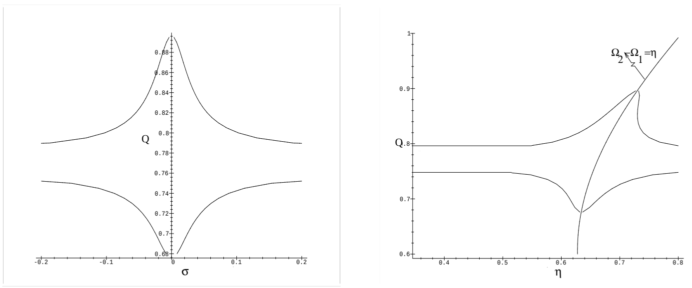
From the linearization of averaged system at the trivial solution we have the characteristic equation in the form $\lambda^4 + q_1\lambda^3 + q_2\lambda^2 + q_3\lambda + q_4 = 0$, in which q_1, q_2, q_3 and q_4 depend on the parameters.

Note that we have $Q_{12} < 0$ and $Q_{21} < 0$. The linear damping coefficients θ_{11} and θ_{22} have a positive sign if $\bar{\beta}V^2 - \bar{\kappa}_2 < 0$; in this case there is no self-excitation. In the case of self-excitation $\bar{\beta}V^2 - \bar{\kappa}_2 > 0$, there are three conditions for θ_{11} and θ_{22}: $\theta_{11} < 0$ and $\theta_{22} > |\theta_{11}|$, $\theta_{22} < 0$ and $\theta_{11} > |\theta_{22}|$, and both of θ_{22} and θ_{11} are positive. The signs of the linear damping coefficients θ_{11} and θ_{22} are important to determine conditions under which the vibrations can be suppressed. In Figure 2 we show the boundaries when the θ_{11} and θ_{22} change sign.

Applying the Routh-Hurwitz criterion to get conditions when the real parts of the eigenvalues have a negative sign leads to two conditions that must be met. The first condition of the Routh-Hurwitz criterion gives $\theta_{11} + \theta_{22} > 0$. The second condition gives the relation $p_1\bar{\sigma}^4 + p_2\bar{\sigma}^2 + p_3 > 0$ where p_j, $j = 1, 2, 3$ depend on Q, M, if the other parameters are fixed. Solving at the boundary, we obtain the interval of stability of the trivial solution is determined by $\eta_\circ + \epsilon\bar{\sigma}_2 < \eta < \eta_\circ + \epsilon\bar{\sigma}_1$.

6. Stability of The Trivial Solution

The parametric excitation is used in the case when self-excited vibrations occur. In the coupled system the effectiveness depends on conditions of the parameter damping θ_{11} or θ_{22}. When both of θ_{11} and θ_{22} are positive, this represents the case where the dynamic absorber successfully cancels the self-excited vibration. This happens in region III in Figure 2.



(a) (b)

FIGURE 3. Stability boundaries for fixed $\varepsilon = \kappa_1 = \beta = 0.2$, $\kappa_2 = 0.1$, $V = \sqrt{2.1}$, $\gamma = 4$ and $M = 0.2$. (a) In the $(\bar{\sigma}, Q)$-plane, (b) in the (η, Q)-plane. Inside the curves in (a) and (b) the trivial solution is stable (full vibration suppression) and it is unstable outside.

In Figure 2, within the small area IV to the left of line $M = M^\star$, both of θ_{11} and θ_{22} are negative. There we have that self-excitation is dominant and full vibration quenching is not possible at all. The firts condition is satisfied on the right side of the line $M = M^\star$.

Note that the region of full vibration suppression in Figure 3 depends on the mass ratio M. The excitation frequency η has a wider range than the frequency ratio Q. Near the combination resonance $\eta = \eta_\circ$ or $\sigma = 0$, the enlargement is increasing with higher values of M, but it does not increase proportionally with η.

Figure 4 shows the influence of the amplitude ε of the parametric excitation on the suppressing area. The parameter mainly influences the size of the area near the combination resonance $\eta = \Omega_2 - \Omega_1$.

The area of suppressing increases with increasing amplitude ε, indicating that ε is a very effective parameter to obtain a large area of vibration suppression.

We point out that this study of stability of system (4.1) is for the realistic case of mass ratio M smaller than 1. For a fixed value of M in this interval we obtain the shapes along the combination resonance and the area as shown in Figure 3. In the numerical simulation shown in [Ecker and Tondl (2000)], this area along the combination resonance is splitting up for small M. In section 7 we explain this analytically by a second order approximation.

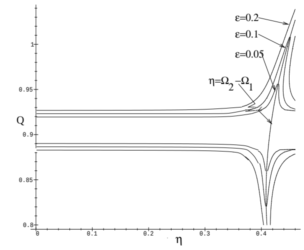


FIGURE 4. Stability boundaries of the trivial solution in the (η, Q)-plane for fixed values of the parameter and varying ε and $M = 0.65$.

7. THE CASE M ORDER ε

In applications we usually have to take the absorber mass (and so the mass ratio M) really small and the question rises whether we can still suppress or at least significantly reduce self-excited vibrations in this case. In [Ecker and Tondl (2000)] a numerical simulation is given which does not agree with the harmonic balance result of the authors. We shall show that this is caused by the necessity to rescale the parameter M with as a consequence that we have to take into account second order effects. We rescale $M = \varepsilon \bar{M}$. In the case the mass ratio M is small; to avoid linear resonance we omit the case $Q \neq 1$ and the combination resonance takes place if $\eta_\circ = |\Omega_2 - \Omega_1|$.

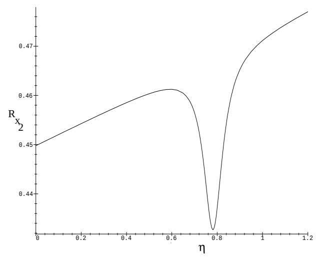


FIGURE 5. The maximum amplitude R_{x_2} of system (3.1) for fixed $Q = 1.105$. The minimum value can be reached at $\eta = 0.77607$ and $R_{x_2} = 0.441621$.

To get a standard form (3.2), we use transformation (3) where now

$$\begin{cases} \text{for} \quad 1 < Q \\ a_1 = 0 \quad \text{and} \quad a_2 = \frac{Q^2-1}{Q^2} \end{cases} \quad \text{or} \quad \begin{cases} \text{for} \quad 0 < Q < 1 \\ a_1 = \frac{Q^2-1}{Q^2} \quad \text{and} \quad a_2 = 0 \end{cases}$$

We find that to the lowest order in ε the trivial solution of the system (for both cases) is unstable. Up to first order in ε, the system has a non-trivial fixed point corresponding with a periodic solution of the original system. The characteristic equation of the linearization at that point is in the form $\lambda^3 + p\lambda^2 + q\lambda + r = 0$. We find that all the coefficients of the equation are positive and $pq - r > 0$,so that the fixed point is stable. Adding the second order ε terms, the fixed point of system up to the lowest order of ε is also stable. The amplitude of variable x_2 of system (3.1). This amplitude will reach the minimum value at $\bar{\sigma}_i = \frac{1}{3}\frac{(\pm 3\gamma_1 - \sqrt{3}\bar{\kappa_1}\eta_\circ)}{2(1-Q)}$. We show that the minimum value at $\eta = 0.77607$ is 0.441621 for fixed $Q = 1.105$, see Figure 5.

8. Conclusion

We have studied system (4.1), modeling flow-induced vibrations, by using the averaging method. There are two conditions needed for suppressing self excited vibrations. The first condition evaluates that the sum of the negative and the positive linear damping components determine the stability of certain modes and must be positive. The second condition is related to the parametric excitation frequency and determines, whether full quenching can be achieved or not in a certain interval. The presented results also demonstrate that a dynamic absorber with parametric excitation is capable of enlarging the range of full vibration suppression near the combination resonance frequency.

For applications the case of a small absorber mass (small M) is important. If M is of order ε the absorber influences the vibration in second order approximation. We find the areas where the vibration is decreased. We can also calculate the minimum value that can be reached by the maximum amplitude of system (3.1) which shows that a large amount of quenching is still possible.

9. Acknowledgments

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References

[Tondl, A. (1959)] . The Method for Determination of Instability Intervals of Quasi-harmonic (parametric) System (in Czech). *Aplikace matematiky*, **4**:278-289.

[Tondl, A.(1978)] . On the Interaction between Self-excited and Parametric Vibrations. *Monographs and Memoranda*,**25**, National Research Institute for Machine Design, Prague.

[Sanders, J.A., and Verhulst, F. (1985)] . *Averaging Methods in Nonlinear Dynamical Systems*, Appl.math. Sciences 59, Springer-Verlag, New York.

[Tondl, A.(1991)] , *Quenching of Self-excited Vibrations*, Elsevier, Prague.

[Tondl, A. (1997)] . To the interaction of different types of excitations. In *Proc.of Sem. Interactions and Feedback 97*, Prague, **25-26**: 111-118.

[Tondl, A. (1998)] . To the problem of quenching self-excited vibrations. *Acta Technica CSAV*, **43**:109-116.

[Ecker, H., and Tondl, A.(2000)] . Suppression of flow-induced vibrations by a dynamic absorber with parametric excitation. *Proc. of 7th International Conference on Flow-Induced Vibrations FIV2000*, Lucerne, Switzerland.

[Tondl, A., Kotek, V., and Kratochvil, C.(2001)] . *Vibration Quenching of Pendulum Type Systems by means of Absorber.* CERM akademicke, Czech Republic.

[Fatimah, S. (2002)] . Bifurcations in dynamical systems with parametric excitation. Ph.D Thesis, Utrecht University.