ENERGETIKA GELOMBANG

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SUB POKOK BAHASAN

A. ENERGI KINETIK DAN ENERGI POTENSIAL
B. PENJABARAN PERSAMAAN GELOMBANG MELALUI KEKEKALAN ENERGI
C. RAPAT ENERGI DAN INTENSITAS GELOMBANG
D. RAPAT MOMENTUM DAN RAPAT ARUS MOMENTUM
E. IMPEDANSI GELOMBANG
F. PEMANTULAN DAN TRANSMISI GELOMBANG

A. ENERGI KINETIK DAN ENERGI POTENSIAL

Energi kinetik terjadi karena gerak massa: $$E_k = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

Energi potensial karena elastisitas: $$E_p = \frac{1}{2}k\left[\psi(x+\Delta x) - \psi(x)\right]^2$$

Ekspansi ke deret Taylor

$$E_p = \frac{1}{2}k\left[\psi(x) + \Delta x\frac{\partial \psi}{\partial x} - \psi(x)\right]^2$$

$$E_p = \frac{1}{2}k(\Delta x)^2\left[\frac{\partial \psi}{\partial x}\right]^2$$

$$v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{k\Delta x}{\Delta m/\Delta x}} = \sqrt{\frac{k(\Delta x)^2}{\Delta m}}$$

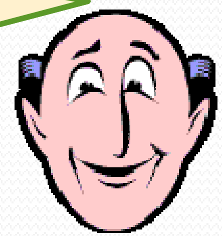
$$k(\Delta x)^2 = \Delta m v^2$$

$$E_p = \frac{1}{2}\Delta m v^2\left[\frac{\partial \psi}{\partial x}\right]^2$$

$$\frac{\partial \psi}{\partial x} = -\frac{1}{v}\frac{d\psi}{dt}$$

$$E_p = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

Apa kesimpulannya?



B. PENJABARAN PERSAMAAN GELOMBANG MEALUI KEKEKALAN ENERGI

$$E_k = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2 \Rightarrow dE_k = \frac{1}{2}dm\left(\frac{\partial \psi}{\partial t}\right)^2 = \frac{1}{2}\rho\, dx\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E_p = \frac{1}{2}k(\Delta x)^2\left[\frac{\partial \psi}{\partial x}\right]^2 \quad E_p = \frac{1}{2}K\Delta x\left[\frac{\partial \psi}{\partial x}\right]^2 \Rightarrow dE_p = \frac{1}{2}K dx\left[\frac{\partial \psi}{\partial x}\right]^2$$

maka total energinya: $dE = dE_k + dE_p \Rightarrow E = \int_0^{n\lambda}\frac{1}{2}\left[\rho\left(\frac{\partial \psi}{\partial t}\right)^2 + K\left(\frac{\partial \psi}{\partial x}\right)^2\right]dx$

Berdasarkan kekekalan energi (energi total = tetap), maka $\frac{dE}{dt} = 0$

$$\frac{1}{2}\int\left\{\rho\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial t}\right)^2 + K\frac{\partial}{\partial t}\left(\frac{\partial \psi}{\partial x}\right)^2\right\}dx = 0 \Rightarrow \frac{1}{2}\int\left\{2\rho\frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2} + 2K\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial t \partial x}\right\}dx = 0$$

$$\Rightarrow \int \rho\frac{\partial \psi}{\partial t}\frac{\partial^2 \psi}{\partial t^2}dx + \int K\frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial t \partial x}dx = 0$$

Kita tahu

$$\frac{d}{dx}\left(\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}\right)=\frac{\partial\psi}{\partial t}\frac{\partial^2\psi}{\partial x^2}+\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial x\partial t}$$

maka

$$\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial t\partial x}=\frac{d}{dx}\left(\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}\right)-\frac{\partial\psi}{\partial t}\frac{\partial^2\psi}{\partial x^2}$$

$$\int\rho\frac{\partial\psi}{\partial t}\frac{\partial^2\psi}{\partial t^2}dx+\int K\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial t\partial x}dx=0$$

$$\int\rho\frac{\partial\psi}{\partial t}\frac{\partial^2\psi}{\partial t^2}dx+\underbrace{K\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}\Bigg|_0^{n\lambda}}_{=0}-\int K\frac{\partial^2\psi}{\partial x^2}\frac{\partial\psi}{\partial t}dx=0$$

$$\psi(x)=A\cos(kx-\omega t)$$

$$\frac{\partial\psi}{\partial x}=-kA\sin(kx-\omega t)$$

$$\frac{\partial\psi}{\partial t}=\omega A\sin(kx-\omega t)$$

$$\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}=-k\omega A^2\sin^2(kx-\omega t)$$

$$x=0\rightarrow n\lambda,\Rightarrow\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial t}=-k\omega A^2\left(\sin^2(n2\pi-\omega t)-\sin^2(-\omega t)\right)=0$$

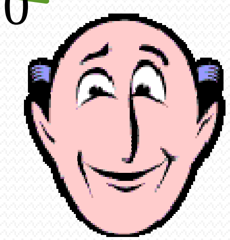
$$\int\left(\rho\frac{\partial^2\psi}{\partial t^2}-K\frac{\partial^2\psi}{\partial x^2}\right)\frac{\partial\psi}{\partial t}dx=0$$

$$\rho\frac{\partial^2\psi}{\partial t^2}-K\frac{\partial^2\psi}{\partial x^2}=0$$

$$\frac{\partial^2\psi}{\partial t^2}-\frac{K}{\rho}\frac{\partial^2\psi}{\partial x^2}=0$$

Merupakan persamaan differensial gelombang.

Sama dengan hasil yang diperoleh pada pembahasan dinamika gelombang.



C. RAPAT ENERGI DAN INTENSITAS GELOMBANG

$$E = E_k + E_p$$

$$E_k = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E_p = \frac{1}{2}\Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E = \Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E = \Delta m\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$E = \Delta m v^2\left(\frac{\partial \psi}{\partial x}\right)^2$$

Sehingga Rapat energi dapat dirumuskan:

$$\varepsilon = \rho v^2\left(\frac{\partial \psi}{\partial x}\right)^2$$

Untuk gelombang dengan persamaan: $\psi(x,t) = \psi_0 \cos(kx - \omega t)$

Maka :

$$\varepsilon = \rho\omega^2\psi_0{}^2 \sin^2(kx - \omega t)$$

$$\varepsilon = \frac{1}{2}\rho\omega^2\psi_0{}^2\{1 - \cos 2(kx - \omega t)\}$$

Karena nilai rata-rata dari $\langle \sin^2(kx - \omega t)\rangle = \frac{1}{2}$

$$\bar{\varepsilon} = \frac{1}{2}\rho\omega^2\psi_0{}^2$$

energi merambat dengan kecepatan v = ω/k juga, dan berubah-rubah secara periodik dengan frekuensi sudut **2ω**

$$\varepsilon = \rho v^2 \left(\frac{\partial \psi}{\partial x}\right)^2$$

Dari rapat energi ini, dapat diturunkan rapat daya persatuan luas penampang atau rapat arus energi (Intensitas)

Untuk gelombang dengan persamaan: $\psi(x,t) = \psi_0 \cos(kx - \omega t)$

Maka :

$$\varepsilon = \rho \omega^2 \psi_0{}^2 \sin^2(kx - \omega t)$$

$$I = \varepsilon v \Rightarrow I = v \rho \omega^2 \psi_0{}^2 \sin^2(kx - \omega t)$$

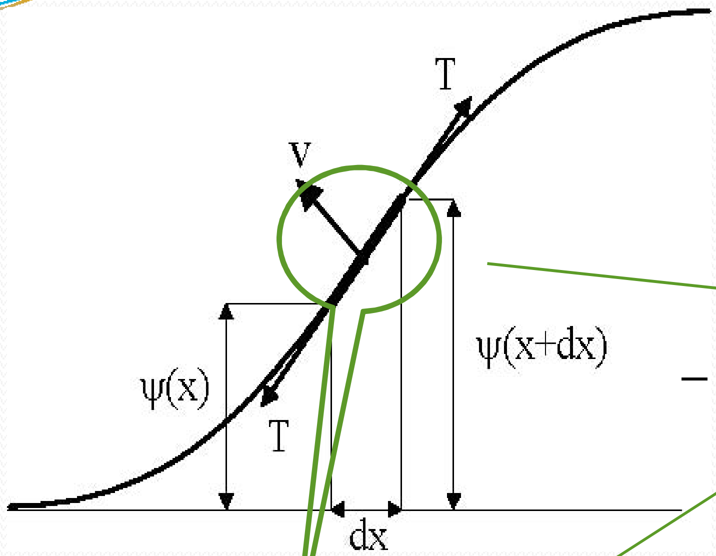
Dari rapat energi rata-rata

$$\bar{\varepsilon} = \frac{1}{2} \rho \omega^2 \psi_0{}^2$$

Dapat diturunkan rapat daya rata-rata persatuan luas penampang atau rapat arus energi rata-rata (Intensitas rata-rata)

$$\bar{I} = \bar{\varepsilon} v \Rightarrow \bar{I} = \frac{1}{2} \rho \omega^2 \psi_0{}^2 v$$

D. RAPAT MOMENTUM

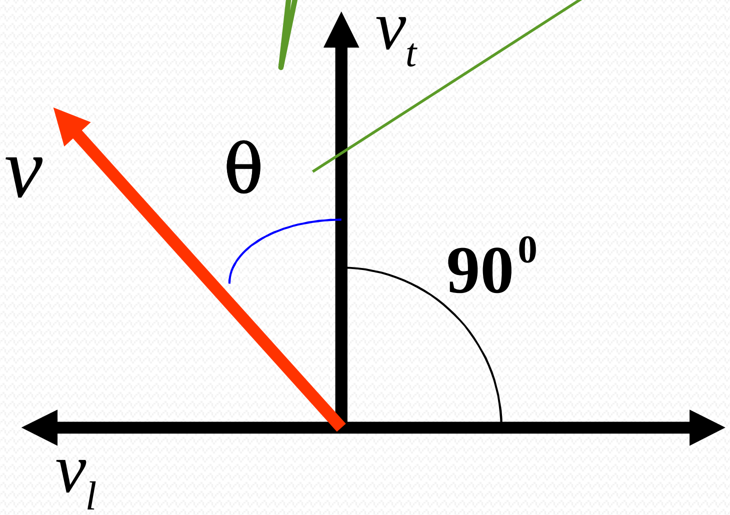


Rapat momentum = momentum persatuan volume

$$p = \frac{dm}{dV} v_l \quad \text{atau} \quad p = \rho v_l$$

$$-\frac{v_l}{v_t} = \tan\theta = \frac{\partial \psi}{\partial x} \quad \text{atau} \quad v_l = -v_t \tan\theta = -v_t \frac{\partial \psi}{\partial x}$$

$$\frac{\partial \psi}{\partial x} = -\frac{1}{v}\frac{\partial \psi}{\partial t} \qquad v_l = -\frac{\partial \psi}{\partial t}\frac{\partial \psi}{\partial x}$$



$$v_l = \frac{1}{v}\left(\frac{\partial \psi}{\partial t}\right)^2 \Leftarrow v_l = -\frac{\partial \psi}{\partial t}\left(-\frac{1}{v}\frac{\partial \psi}{\partial t}\right)$$

$$p = \rho v_l \Rightarrow p = \frac{\rho}{v}\left(\frac{\partial \psi}{\partial t}\right)^2$$

rapat momentum

$$p = \frac{\rho}{v}\left(\frac{\partial \psi}{\partial t}\right)^2$$

Dari rapat momentum ini, dapat diturunkan rapat aliran momentum persatuan waktu atau rapat arus momentum g

$$g = pv$$

Jadi gelombang mengangkut daya persatuan luas penampang (aliran rapat energi = intensitas), dan momentum (dinyatakan oleh aliran rapat momentum persatuan waktu).

substitusi

$$g = \frac{\rho}{v}\left(\frac{\partial \psi}{\partial t}\right)^2 v$$

$$\varepsilon = pv$$

Rapat Energi

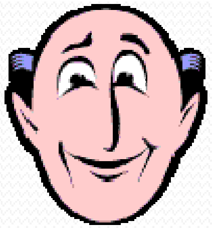
$$\varepsilon = \rho v^2\left(\frac{\partial \psi}{\partial x}\right)^2$$

$$g = \rho\left(\frac{\partial \psi}{\partial t}\right)^2$$

$$g = \varepsilon$$

$$\varepsilon = \rho\left(\frac{\partial \psi}{\partial t}\right)^2$$

Rapat Arus Momentum



Untuk gelombang dengan persamaan: $\psi(x,t) = \psi_0 \cos(kx - \omega t)$

Intensitas: $I = \varepsilon\, v$ atau $I = g\, v$

Intensitas rata-rata: $\bar{I} = \bar{\varepsilon}\, v$ atau $\bar{I} = \bar{g}\, v$

$$\bar{p} = \frac{1}{2}\frac{\rho}{v}\omega^2 \psi_0{}^2$$

$$\bar{g} = \frac{1}{2}\rho\omega^2 \psi_0{}^2$$

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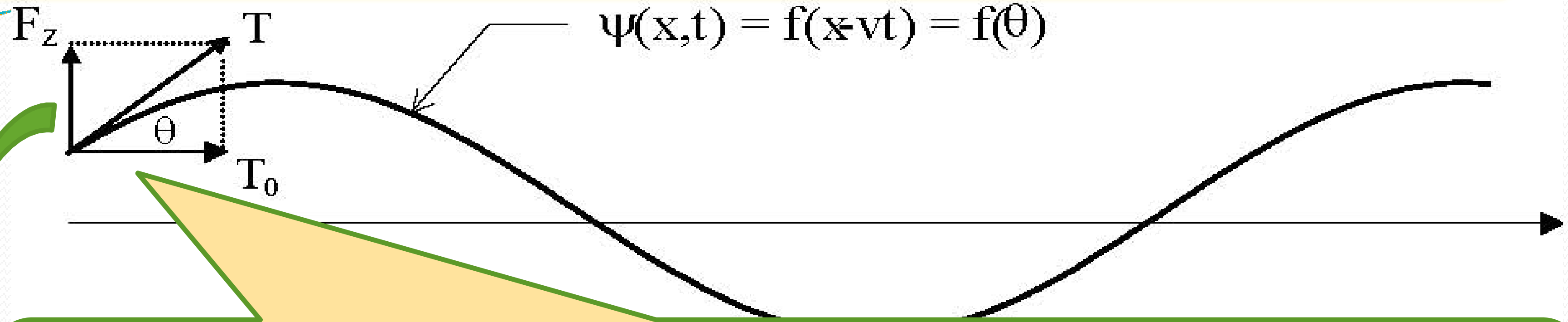
E. IMPEDANSI DAN DAYA GELOMBANG



Tinjau suatu gelombang yang merambat pada bagian tali. Ketika tali mendapat gangguan gaya luar F_Z, karena sifat inersianya, tali akan melawan gaya ini dengan gaya yang sebanding dengan kecepatan.

Besarnya gaya pada tali yang melawan gaya luar F_z sebesar: $F_z = -Z\frac{d\psi}{dt}$

$$F_z = T_0 \tan\theta \qquad F_z = T_0\frac{d\psi}{dx}$$

$$\frac{d\psi}{dx} = -\frac{1}{v}\frac{d\psi}{dt}$$

$$F_z = -\frac{T_0}{v}\frac{d\psi}{dt}$$

Impedansi Gelombang

$$Z = \frac{T_0}{v}$$

$$Z = \sqrt{T_0\,\rho}$$

karena $v = \sqrt{\frac{T_0}{\rho}}$

Daya Gelombang $P = F_z\,v$

$$F_z = Z\frac{d\psi}{dt}$$

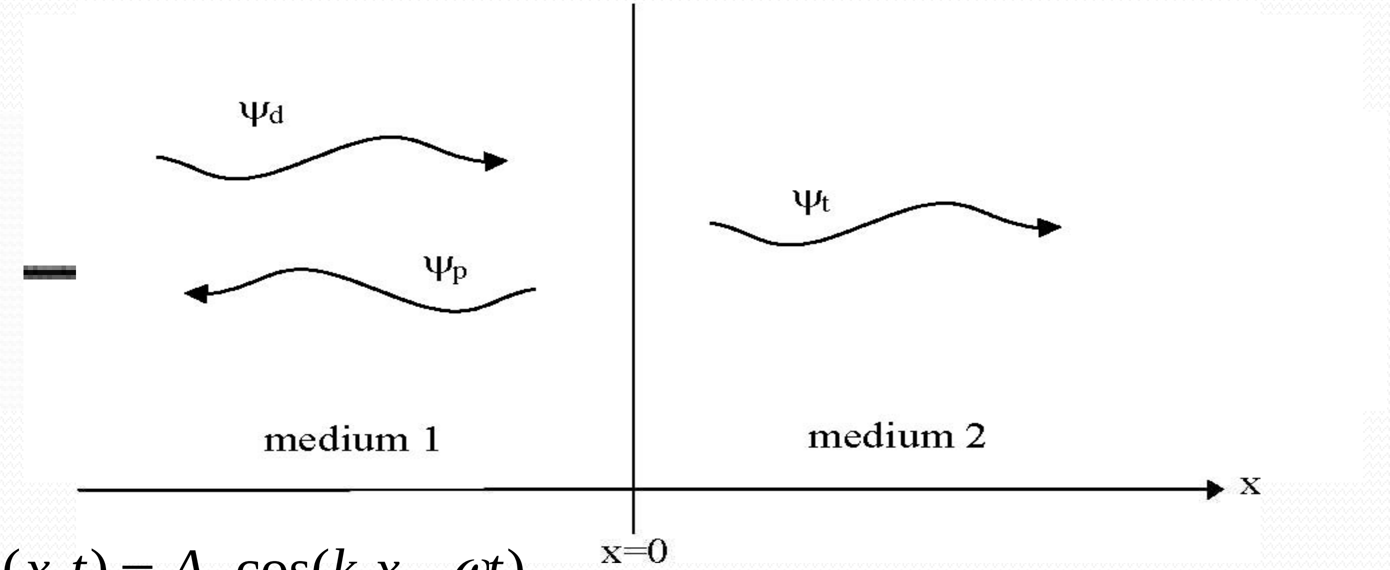
$$v = \frac{d\psi}{dt}$$

$$P = Z\left(\frac{d\psi}{dt}\right)^2$$

mempunyai bentuk yang sama dengan daya pada rangkaian listrik

$$P = Z\left(\frac{dq}{dt}\right)^2$$

F. PEMANTULAN DAN TRANSMISI GELOMBANG



$$\psi_d(x,t) = A_d \cos(k_1 x - \omega t)$$

$$\psi_t(x,t) = A_t \cos(k_2 x - \omega t)$$

$$\psi_p(x,t) = A_p \cos(k_1 x + \omega t)$$

Dari syarat batas (di x = o) kontinuitas simpangan: $\psi_d(0,t) + \psi_p(0,t) = \psi_t(0,t)$

$$A_d + A_p = A_t \rightarrow 1 + \frac{A_p}{A_d} = \frac{A_t}{A_d} \rightarrow 1 + r = t$$

$r = \dfrac{A_p}{A_d}$ = koefisien pantul

$t = \dfrac{A_t}{A_d}$ = koefisien transmisi

Dari syarat batas kontinuitas kemiringan: $\frac{\partial \psi_d(0,t)}{\partial x} + \frac{\partial \psi_p(0,t)}{\partial x} = \frac{\partial \psi_t(0,t)}{\partial x}$

$$k_1(A_d - A_p) = k_2 A_t \rightarrow k_1(A_d - A_p) = k_2(A_d + A_p)$$

$$\Rightarrow (k_1 - k_2)A_d = (k_1 + k_2)A_p \rightarrow \frac{A_p}{A_d} = \frac{k_1 - k_2}{k_1 + k_2}$$

$r = \frac{k_1 - k_2}{k_1 + k_2}$ Sustitusi ke $1 + r = t$ diperoleh $t = \frac{2k_1}{k_1 + k_2}$

Mengingat $k = \frac{\omega}{v}$ dan untuk tali $Z = \frac{T_0}{v}$, maka

$r = \frac{Z_1 - Z_2}{Z_1 + Z_2}$ dan $t = \frac{2Z_1}{Z_1 + Z_2}$

Berkaitan dengan daya, dikenal besaran reflektansi (R) yakni perbandingan daya yang dipantulkan terhadap daya gelombang datang, dan transmitansi (T) yakni perbandingan daya yang ditransmisikan terhadap daya gelombang datang.

$$R = \frac{P_p}{P_d} = \frac{(A_p)^2}{(A_d)^2} \Rightarrow R = r^2$$

$$T = \frac{P_t}{P_d} = \frac{Z_2 (A_t)^2}{Z_1 (A_d)^2} \Rightarrow T = \frac{Z_2}{Z_1} t^2$$

Berdasarkan hukum kekekalan energi: $E_d = E_p + E_t$

$$1 = \frac{E_p}{E_d} + \frac{E_t}{E_d} \Rightarrow 1 = \frac{P_p}{P_d} + \frac{P_t}{P_d} \Rightarrow R + T = 1$$

Tinjau beberapa kasus khusus berikut:

a. Bila $Z_2 = Z_1$ maka r = 0 dan t = 1

Terjadi transmisi total

b. Bila $Z_2 >> Z_1$ maka r = -1 dan t = 0

Terjadi pembalikan fase pada gelombang pantul

Persamaan gelombang pada medium 1 merupakan superposisi dari gelombang datang dan gelombang pantul

$$\psi_1 = A_d\left[\cos(k_1 x - \omega t) - \cos(k_1 x + \omega t)\right]$$

$$\psi_1 = 2A_d \sin(k_1 x)\sin(\omega t)$$

Dari persamaan di samping terlihat bahwa pemantulan total menghasilkan gelombang berdiri dengan amplitude $2A_d\sin(k_1x)$, seperti pada tali dengan ujung terikat.

c. Bila $Z_2 << Z_1$ maka $r = 1$ dan $t = 2$

$$\psi_1 = A_d \cos(k_1 x - \omega t) + A_d \cos(k_1 x + \omega t)$$

$$\psi_1 = 2A_d \cos(k_1 x)\cos(\omega t)$$

Dari persamaan ini terlihat bahwa pemantulan total menghasilkan gelombang berdiri dengan amplitudo $\psi_1 = 2A_d \cos(k_1 x)$ seperti pada tali dengan ujung bebas.

Tinjau kasus umum $Z_2 > Z_1$ dan $Z_2 < Z_1$ berikut:

$Z_2 > Z_1$ (maka $\rho_2 > \rho_1$)

$Z_2 < Z_1$ (maka $\rho_2 < \rho_1$)