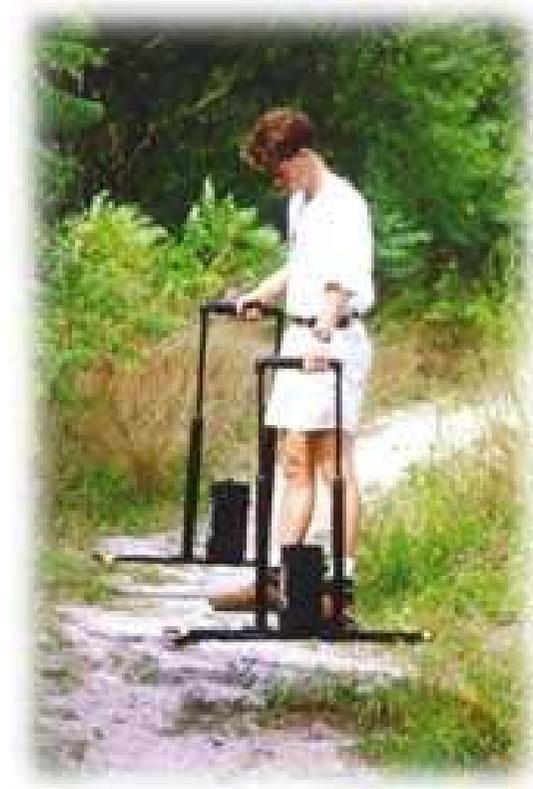
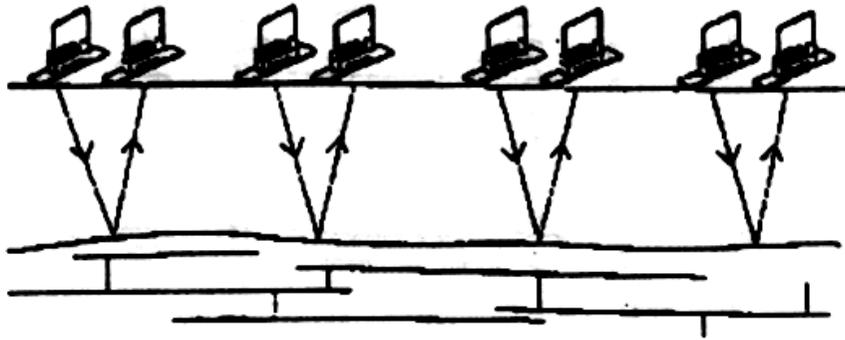


GLE 594: An introduction to applied geophysics

Ground Penetrating Radar

Fall 2005

Ground Penetrating Radar

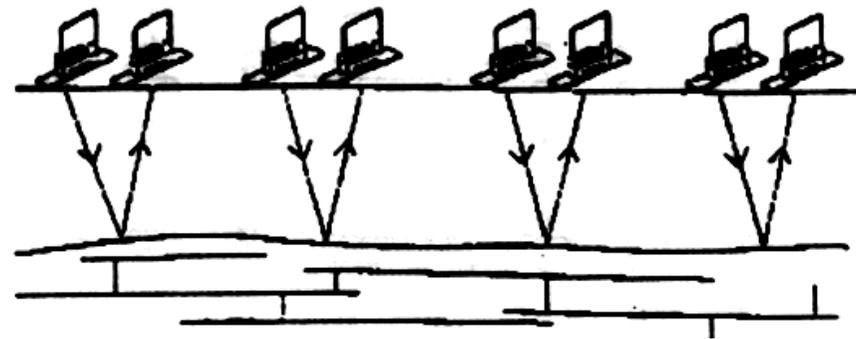


- Reading
 - Today: 309 - 316
 - Next class: 316 - 329

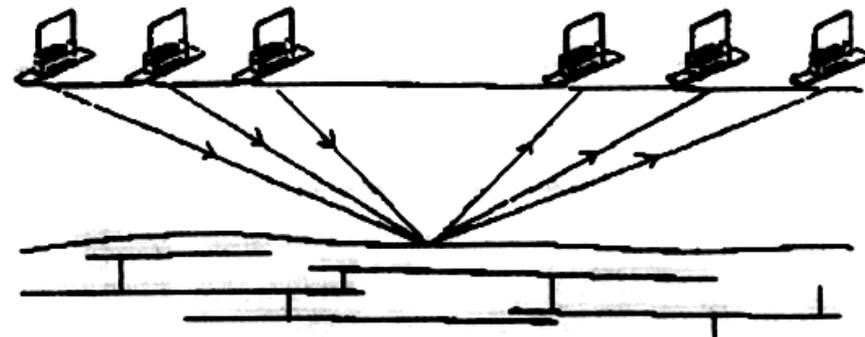
Introduction to GPR

- Using the reflection (and sometimes transmission) of 'high-frequency' EM waves to image the shallow subsurface.
- GPR surveys work best in low conductivity media.

REFLECTION



COMMON MID POINT (CMP)



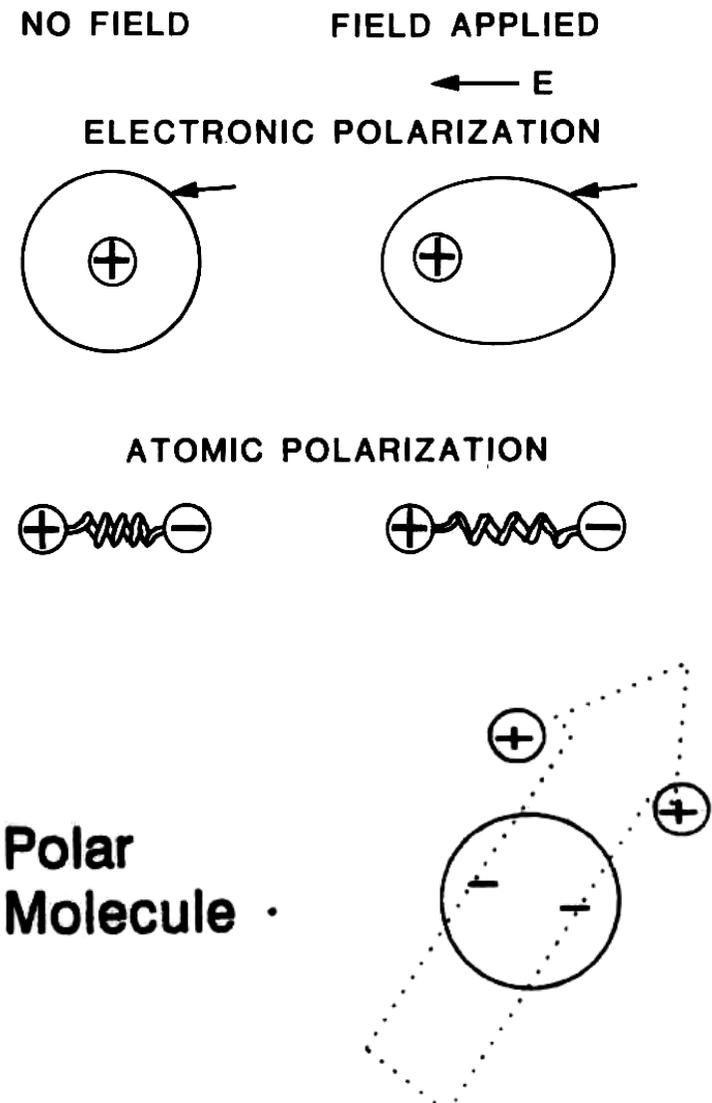
Dielectric Properties

- $\mathbf{D} = \epsilon \mathbf{E}$ where ϵ [F/m] is the dielectric permittivity

$$\epsilon = \epsilon_r \cdot \epsilon_0 = k \cdot \epsilon_0$$

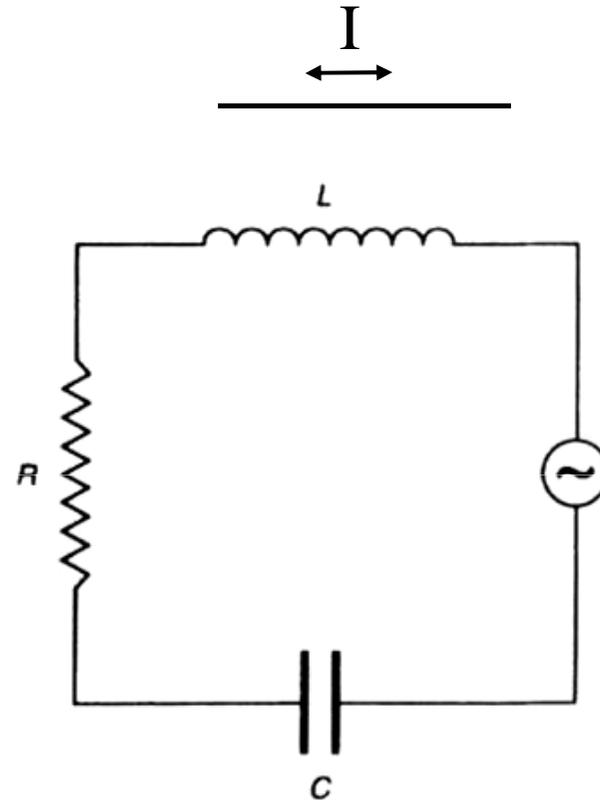
$\epsilon_0 = 8.85 \cdot 10^{-12}$ F/m is the dielectric permittivity of free space

ϵ_r or k is the relative dielectric permittivity (a.k.a. dielectric constant): 1 in air and 80 in water



The GPR 'Earth Circuit'

- In a simplistic sense, at frequencies where the displacement current becomes important the earth can be thought of as an RC circuit
- Why no inductance term?
 - Inductor impedance: $Z_L = i\omega L$
 - Capacitor Impedance: $Z_C = 1/i\omega C$
 - Thus as frequency gets large, $Z_L \Rightarrow \infty$ while $Z_C \Rightarrow 0$.



Dielectric Geomaterials Properties

- Can use mixing laws to describe bulk dielectric constant:
 - Topp's Equation

$$\epsilon_r^b = 3.03 + 9.3 \cdot \theta + 14.6 \cdot \theta^2 - 76.6 \cdot \theta^3$$

where θ is the volumetric moisture content.

- Mixing models

$$\epsilon_{\text{Formation}}^{\beta} = \sum_i \epsilon_{ri}^{\beta} \theta_i$$

- ϵ_i represents dielectric constant of different components.
- θ_i represents volumetric fraction of each component.

Dielectric Properties of Earth Materials

- Typical dielectric constant and corresponding electrical conductivity for various geologic materials
- Note how amount of water controls dielectric constant

MATERIAL	ϵ_r	σ (mS/M)
Air	1	0
Distilled Water	80	0.01
Fresh Water	80	0.5
Sea Water	80	3×10^3
Dry Sand	3-5	0.01
Saturated Sand	20-30	0.1-1.0
Limestone	4-8	0.5-2
Shales	5-15	1-100
Silts	5-30	1-100
Clays	5-40	2-1000
Granite	4-6	0.01-1
Dry Salt	5-6	0.01-1
Ice	3-4	0.01

Revisit E Field in Wave Equation

Assume sinusoidal time dependence:

$$\mathbf{E} = \mathbf{E}_0 e^{-i\omega t}$$

Then

$$\nabla^2 \mathbf{E} + (\mu\epsilon\omega^2 - i\omega\mu\sigma)\mathbf{E} = 0$$

or

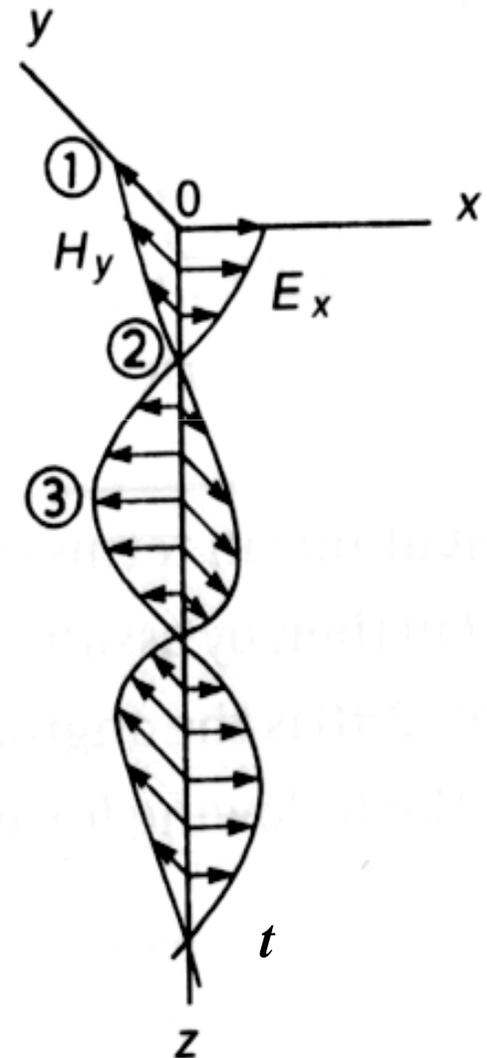
$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0$$

where

$$k = \sqrt{\omega^2 \mu\epsilon - i\sigma\omega\mu} = \beta - i\alpha$$

and

$$\nabla^2 \mathbf{E} = \frac{\partial^2 E_x}{\partial x^2} \hat{\mathbf{x}} + \frac{\partial^2 E_y}{\partial y^2} \hat{\mathbf{y}} + \frac{\partial^2 E_z}{\partial z^2} \hat{\mathbf{z}}$$



Revisit E Field in Wave Equation

- Remember sinusoidal solution to plane-wave equation is:

$$E_x(z, t) = E_0 e^{-i(kz - \omega t)} = E_0 e^{-i(\beta z - \omega t)} e^{-\alpha z}$$

- In the induction regime $\sigma \gg \omega\epsilon$; thus $\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}}$
- In radar, or propagation regime, $\sigma \ll \omega\epsilon$ (low-loss media):

– $\beta = \omega\sqrt{\mu\epsilon}$: related to velocity of propagating wave

- Velocity:
$$V = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$$

- Wavelength: $\lambda = 2\pi/\beta = V/f$

– $\alpha \approx \sqrt{\frac{\mu}{\epsilon}} \frac{\sigma}{2} = \frac{\mu\sigma V}{2}$: related to attenuation of propagating wave

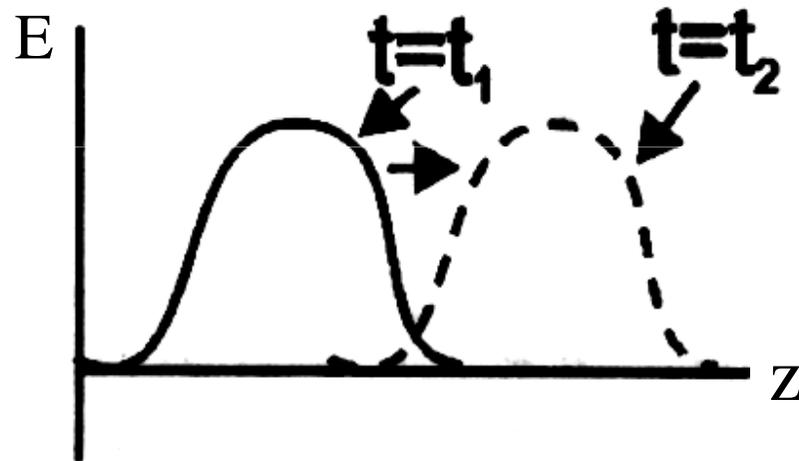
- Skin depth: $\delta = 1/\alpha$

- Lossless media: $\sigma = 0$. Then $\alpha = 0$ and pure propagation

Lossless and Non-Ferromagnetic Medium

- Velocity given as:
$$V = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$$

- If $\sigma=0$, plane wave propagates in z direction with no attenuation.



Low-Loss Media

- Velocity the same as lossless media:

$$\alpha \approx \sqrt{\frac{\mu}{\epsilon}} \frac{\sigma}{2}$$

thus exponential decay of wave amplitude with distance.

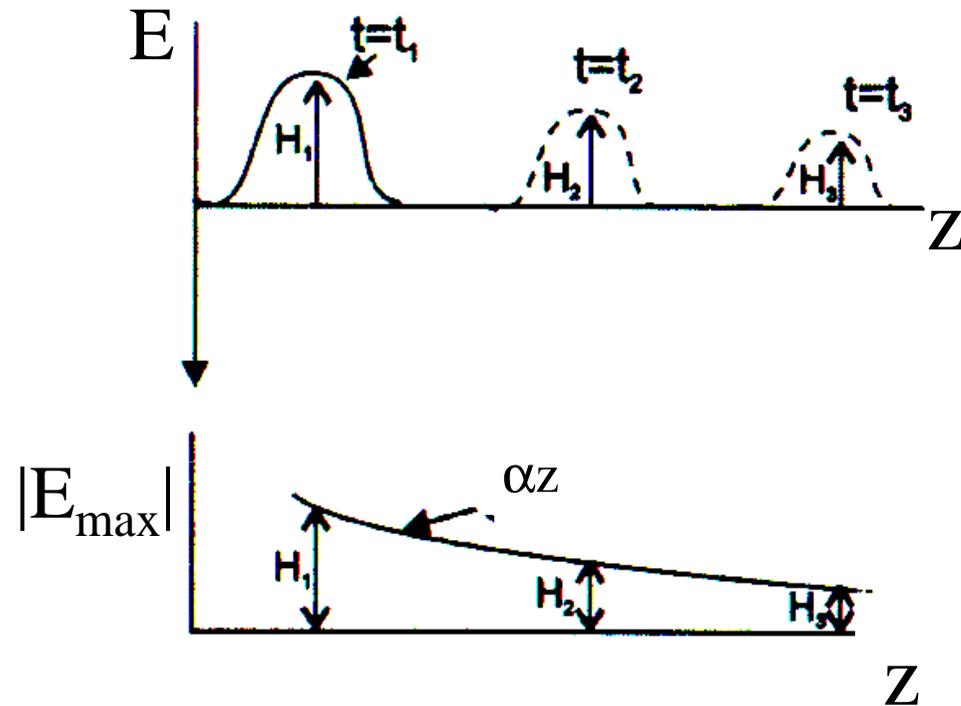
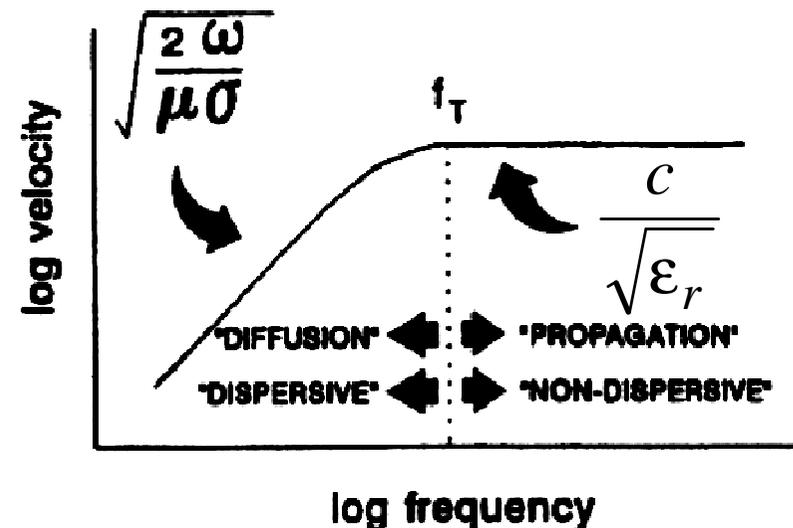
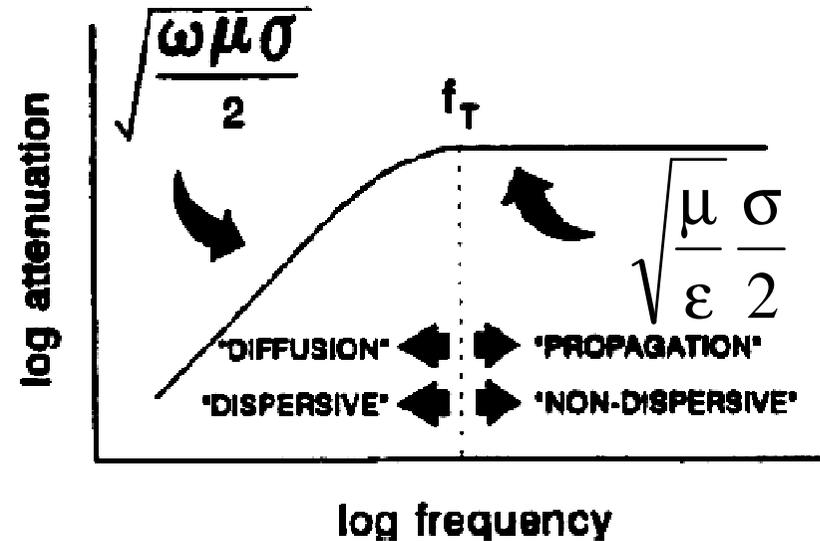


Figure 2.10: EM fields propagate as spatially damped waves when electrical losses are small. The signal amplitude decays exponentially in the direction of field translation while the field shape remains invariant.

Velocity and Attenuation Spectrum

- $f_t = \text{transition frequency}$.
- Dispersion refers to frequency dependent velocity.
- Note: there is a transition region from pure diffusion to pure propagation.



Low-Loss and Lossless Properties

- Typical dielectric constant, electrical conductivity, velocity and attenuation for various geologic materials.

MATERIAL	ϵ_r K	σ (mS/M)	v (m/ns)	α (dB/m)
Air	1	0	0.30	0
Distilled Water	80	0.01	0.033	2×10^{-3}
Fresh Water	80	0.5	0.033	0.1
Sea Water	80	3×10^3	.01	10^3
Dry Sand	3-5	0.01	0.15	0.01
Saturated Sand	20-30	0.1-1.0	0.06	0.03-0.3
Limestone	4-8	0.5-2	0.12	0.4-1
Shales	5-15	1-100	0.09	1-100
Silts	5-30	1-100	0.07	1-100
Clays	5-40	2-1000	0.06	1-300
Granite	4-6	0.01-1	0.13	0.01-1
Dry Salt	5-6	0.01-1	0.13	0.01-1
Ice	3-4	0.01	0.16	0.01

Reflection and Transmission

- Low-loss Reflection

$$R = \frac{V_1 - V_2}{V_2 + V_1} = \frac{\sqrt{\epsilon_{r1}} - \sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}} + \sqrt{\epsilon_{r2}}}$$

- Low-loss Transmission

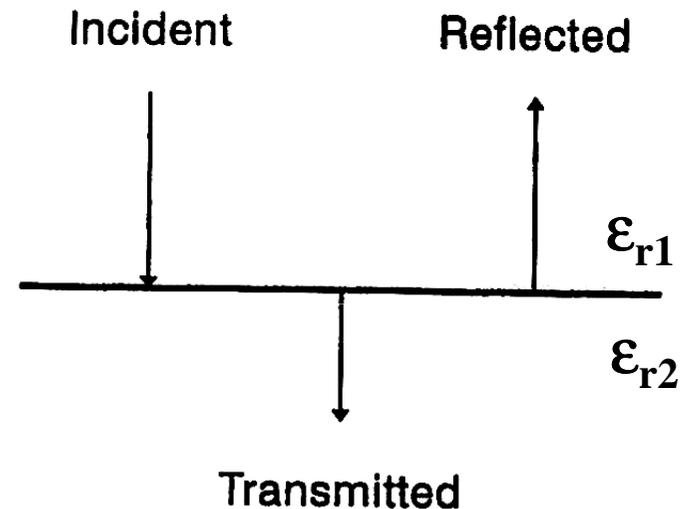
$$T = \frac{2V_1}{V_2 + V_1} = \frac{2\sqrt{\epsilon_{r2}}}{\sqrt{\epsilon_{r1}} + \sqrt{\epsilon_{r2}}}$$

- Reflections off of metal

– $\epsilon\omega \ll \sigma$ and thus $V_2 = (2\omega/\mu\sigma)^{1/2}$

– Results

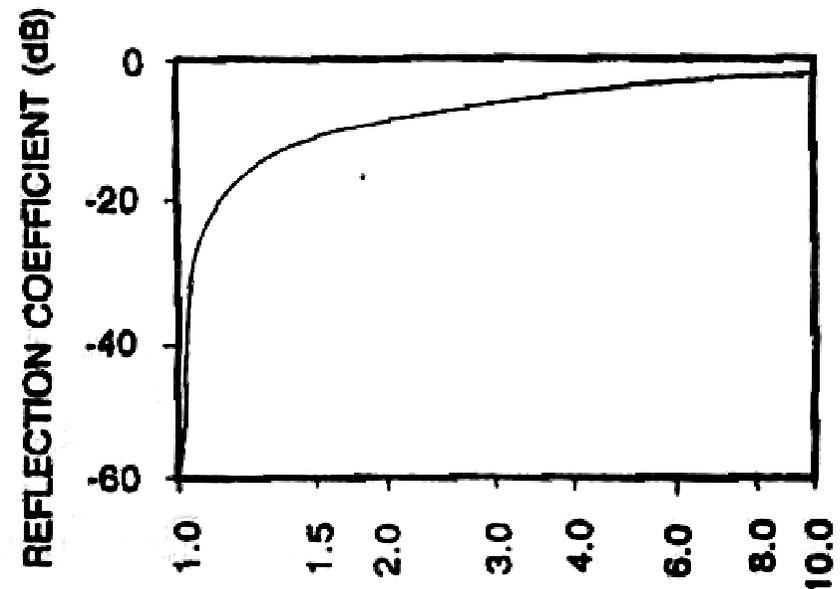
- $R = -1$
- $T \approx 0$



GPR Reflection Coefficients

VERTICAL INCIDENCE REFLECTION COEFFICIENT FOR SOME TYPICAL GEOLOGICAL CONTACTS

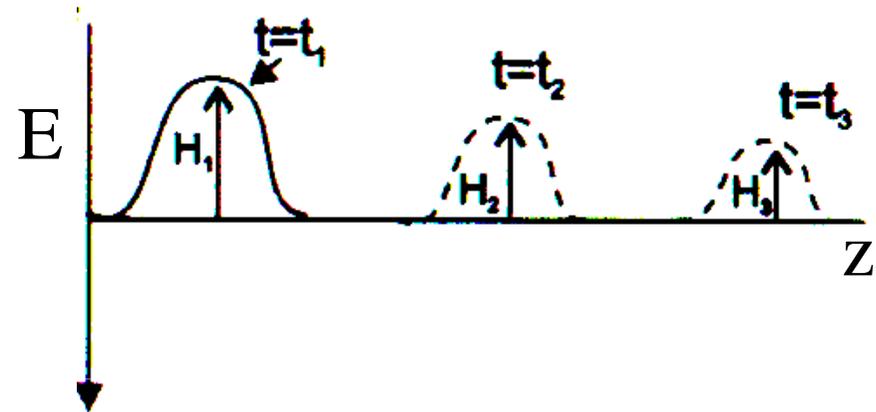
FROM		TO		REFLECTION COEFFICIENT (dB)	
Air	K=1	Dry Soil	K=5	-0.38	-8.4
Dry Soil	K=5	Wet Soil	K=25	-0.38	-8.4
Dry Soil	K=5	Rock	K=8	-0.12	-10
Wet Soil	K=25	Rock	K=8	0.28	-11
Water	K=81	Gyttja	K=50	0.12	-18
Water	K=81	Rock	K=8	0.52	-5.7
Ice	K=3.2	Water	K=81	0.67	-3.5
Frozen Soil	K=6	Wet Soil	K=25	0.34	-9.3
Soil	K=3-50	Metal	K → ∞	-1	0



$$\sqrt{\frac{\epsilon_{r2}}{\epsilon_{r1}}} = \frac{V_1}{V_2}$$

Depth Sensitivity

- Controlled by attenuation
 - ϵ_r may vary by factor of 5 at most.
 - σ may vary by orders of magnitude.
 - Thus attenuation controlled by conductivity.



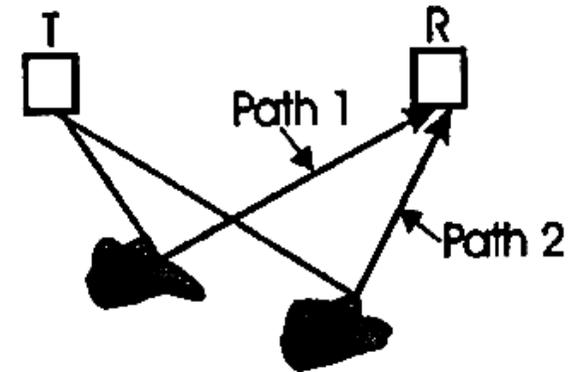
- Signal penetration depth decreases with increasing conductivity.
- good rule of thumb is a target might be detectable if above $0.035/\sigma$ [m].

Instrument Resolution

- Definition of Resolution
 - The ability of the measurement systems to distinguish between two signals, or the minimum separation that two objects can be separated by and still be uniquely imaged.



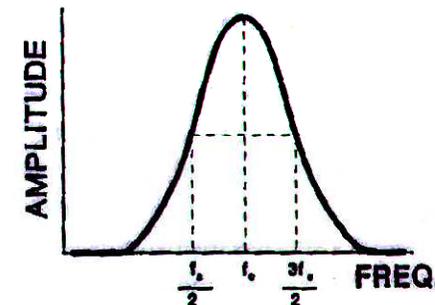
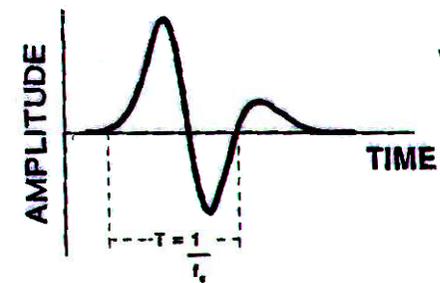
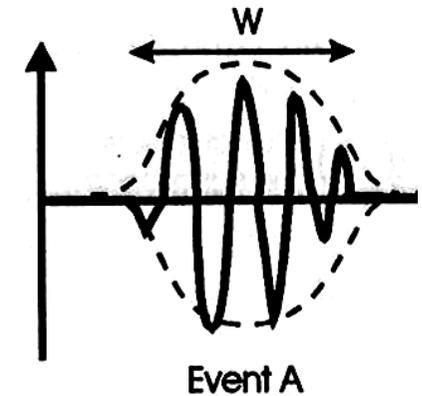
Figure 5.4c: Examining pulse envelopes only, two events are said to be resolved when separated by a time $w/2$.



Instrument Resolution

- Other Definitions

- Pulse width W : refers to the curve in time and amplitude that encloses the pulse.
- Pulse spectrum: Frequency content of the transmitter signal.
- Center frequency: Frequency at which the pulse spectrum maximizes. Usually the center of pulse spectrum.
- Bandwidth: the range of frequencies, or spectrum, over which measurements are made. Inversely proportional to pulse width.



Bandwidth and Center Frequency versus Pulse-Width and Resolution

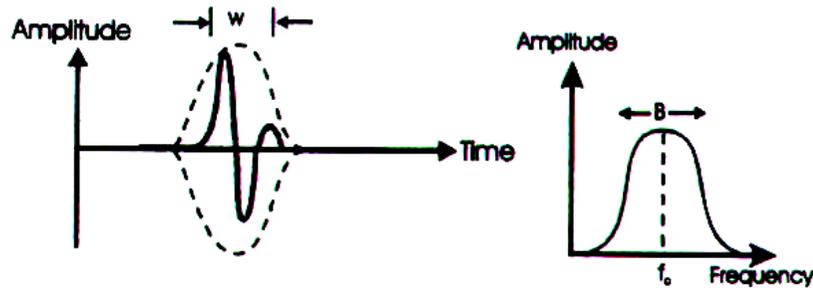


Figure 5.6b: A wide band signal has few if any oscillations, a short time duration and broad frequency content which can at best be said to be centred at a frequency, f_c .

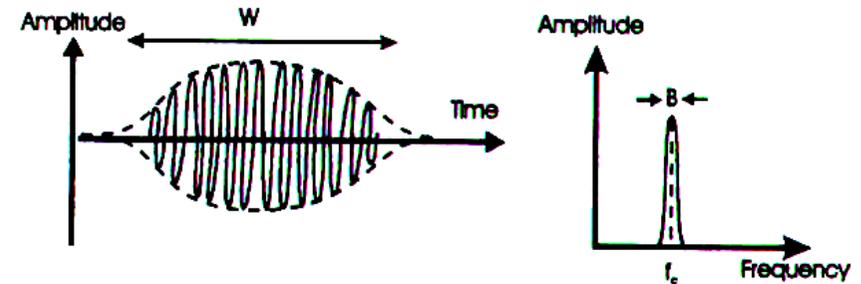
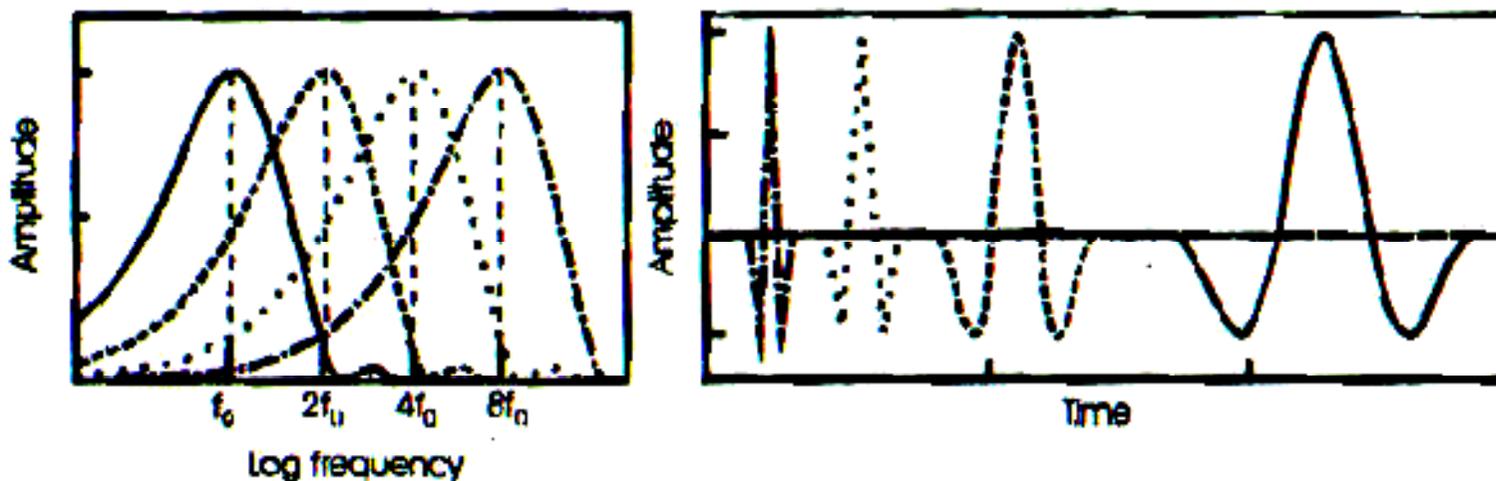
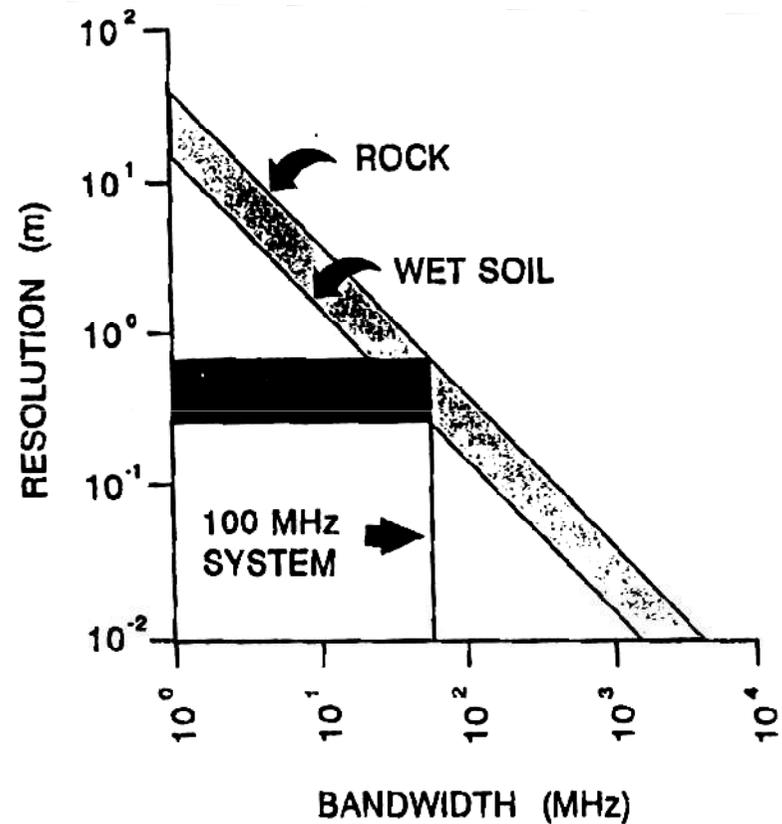


Figure 5.6a: A narrow bandwidth signal is a long oscillatory pulse characterized by single dominant frequency, f_c .



Instrument Requirements

- The best resolution that a system can obtain is controlled by the system bandwidth.
- Generally resolution
 - increases with decreasing pulse width
 - increases with increasing center frequency and bandwidth



Resolution versus Bandwidth

REFLECTOR RESOLUTION

Centre Frequency	Bandwidth (MHz)	Pulsewidth (ns)	Resolution (m)
200.0	200.0	5	0.25
100.0	100.0	10	0.50
50.0	50.0	20	1.00
25.0	25.0	40	2.00
12.5	12.5	80	4.00

Note: Resolution assumes a dielectric constant of 9

Summary of depth, resolution, and bandwidth typically needed in GPR systems

Maximum Depth (m)	Resolution Required(m)	Envelope Width in (ns)	Required Bandwidth (MHz)
0.1	0.001	0.04	2500.0
1.0	0.010	0.40	250.0
10.0	0.100	4.00	25.0
100.0	1.000	40.00	2.5

$$\text{Bandwidth} = \frac{1}{\text{Pulsewidth}}$$

**Note: A material with a permittivity of 9 is assumed in calculated envelope width and bandwidth.*

Resolution Topics

- Resolving two close but separate objects.
 - Time Domain: Two targets can be defined as separate if the distance between them is greater than $\frac{1}{2}$ the pulse width.
 - Frequency domain: two objects or interfaces are uniquely resolvable if the separation between them is at least $\frac{\lambda}{4}$ in theory but $\frac{\lambda}{2}$ in practice

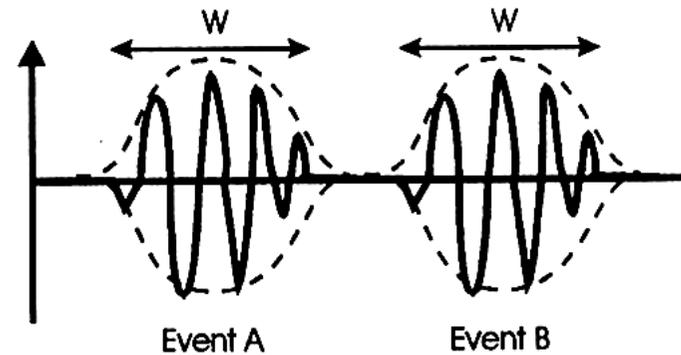


Figure 5.4a: Two radar return events are well resolved if the two transient signals are clearly separated in time as depicted here. The dotted lines, called the pulse envelope, are totally separated.

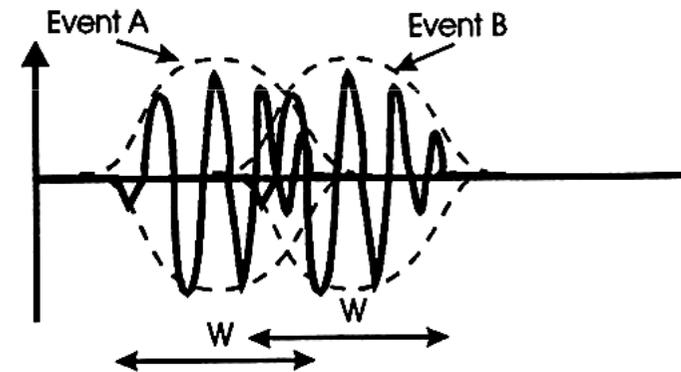


Figure 5.4b: When two events overlap each other in time, the question arises as to whether one or two events are present.



Figure 5.4c: Examining pulse envelopes only, two events are said to be resolved when separated by a time $w/2$.

Resolution Topics

- Transmitter blanking - Inability of receiver to detect reflected signal until after transmitter is off, or air wave has passed.

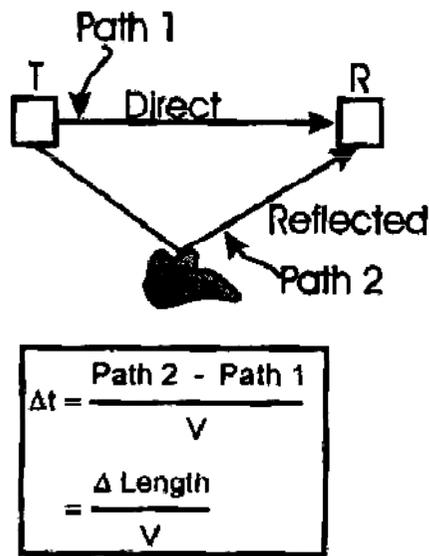
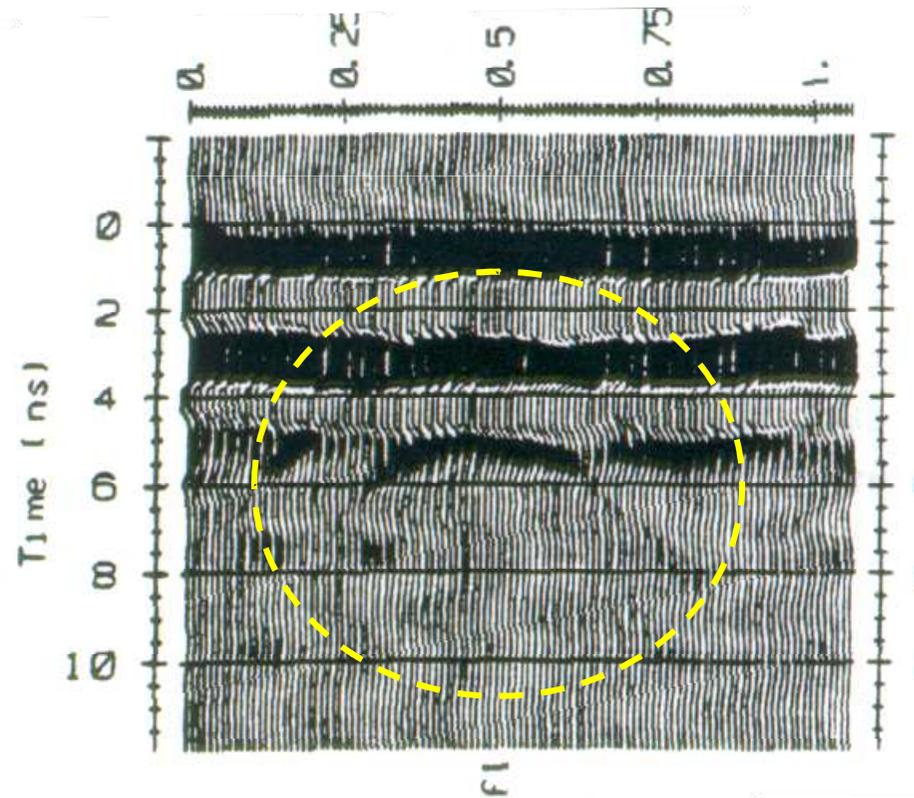
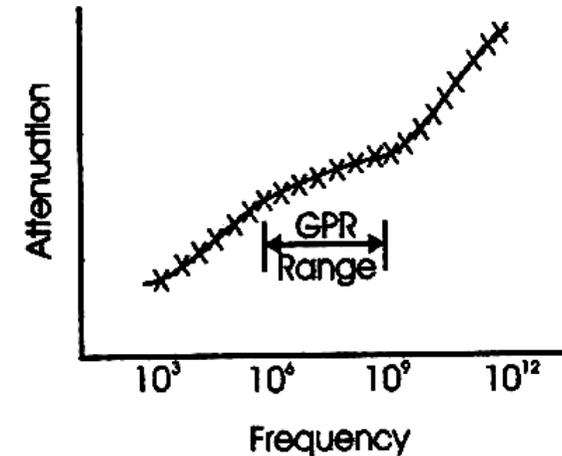
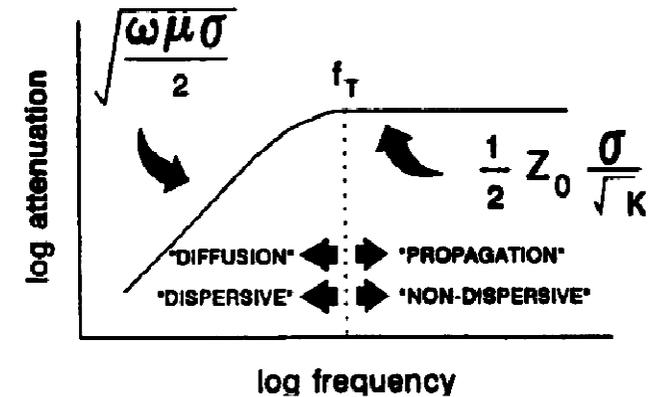


Figure 5.5a: Transmitter blanking occurs when the direct signal travelling from the transmitter to the receiver overlaps in time reflected signals.



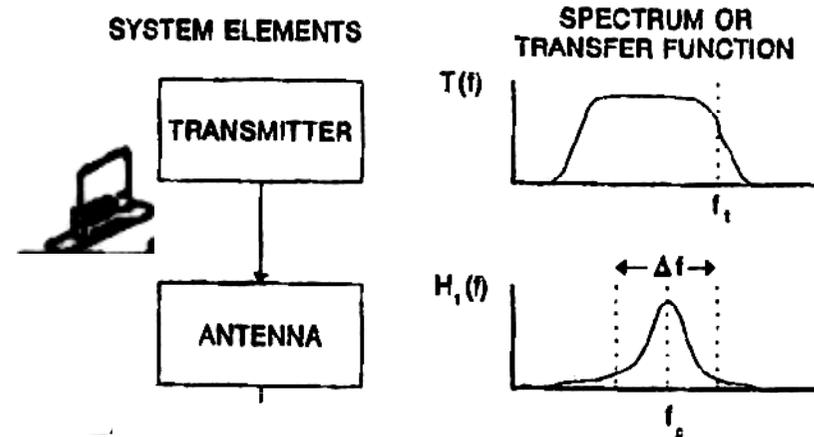
Resolution versus Depth

- If in true low-loss regime, resolution will be somewhat independent with depth. Why?
 - Velocity and more importantly attenuation are frequency independent.
- When low-loss condition is not met, attenuation is frequency dependent.
 - Depth-dependent resolution is then empirical.



GPR System and Signal Generation

- Transmitter generates very short pulses of EM energy (1 to 10's of ns). How times are determined?
 - Depths of targets
 - Shallow as a few cm
 - Deep as tens of m
 - Different velocities
 - min velocity 0.033m/ns (water)
 - max velocity 0.3 m/ns (air)
 - Combining yields
 - min depth with max velocity produces a min time of ~50 pico-sec
 - max depth with min velocity produces max time of ~10 μ sec
 - Center frequency and bandwidth controlled by antennas.
 - Radiated power maximizes for $\lambda/4$ in size
 - Thus antenna length larger for lower frequencies
 - Wave then travels downward and outward in the Earth.



GPR System and Signal Generation

- Wave encounters regions of contrasting properties generating reflections, refractions and diffractions.
- Reflected, diffracted and refracted waves detected by receiver.
- Procedure repeated to reduce random noise, then antennas moved to new position.

