# Investigating a mass-on-spring oscillator

# Demonstration

A mass suspended on a spring will oscillate after being displaced. The period of oscillation is affected by the amount of mass and the stiffness of the spring. This experiment allows the period, displacement, velocity and acceleration to be investigated by datalogging the output from a motion sensor. It is an example of simple harmonic motion.

# **Apparatus and materials**

- Motion sensor, interface and computer
- Slotted masses on holder, 100 g 400 g
- Clamp and stand
- String
- Springs, 3
- Card

# **Technical notes**



Suspend the spring from a clamp and attach a mass to the free end. Adjust the height of the clamp so that the mass is about 30 cm above the motion sensor which faces upwards.

The clarity of measurements depends upon the choice of spring stiffness and mass. Good

results can be obtained with three springs linked in series, and masses in the range 100 - 400 g. With this choice, it is necessary to place the sensor on the floor and allow the mass and spring to overhang the edge of the bench.

When the mass is displaced and released, its vertical motion is monitored by a motion sensor connected via an interface to a computer.

In general, the magnitude of the initial displacement should not exceed the extension of the spring. It is best to lift the mass to displace it, rather than pull it down.

The mass may acquire a pendulum type of motion from side to side. Eliminate this by suspending the spring from a piece of string up to 30 cm.

For the collection and analysis of the data, data-logging software is required to run on the computer. Configure the program to measure the distance of the mass from the sensor, and to present the results as a graph of distance against time. Scale the vertical axis of the graph to match the amplitude of oscillation.

# Safety

Unless the stand is very heavy, use a G-clamp to anchor it to the bench.

#### Procedure

#### **Data collection**

**a** Lift and release a 400 g mass to start the oscillation. Start the data-logging software and observe the graph for about 10 seconds.

**b** Before the oscillation dies away, restart the data-logging software and collect another set of data which can be overlaid on the first set.

c Repeat the experiment with 300 g, 200 g and 100 g masses.

#### Analysis

#### Measurement of period

**d** The period of the sinusoidal graph may be measured using a time interval analysis tool in the software. Measure the period from peak to peak.

**e** Take measurements at several different places on the time axis, and observe that the period does not vary with elapsed time.

**f** Take similar measurements on the set of results with a smaller amplitude, and observe that the period appears to be independent of amplitude.

Effect of mass

**g** Measure the period for each of the other graphs resulting from using different masses. Plot a new graph of period against mass. (Y axis: period; X axis: mass).

**h** Use a curve-matching tool to identify the algebraic form of the relationship. This is usually of the form 'period is proportional to the square root of mass'.

**i** Use the program to calculate a new column of data representing the square of the period. Plot this against mass on a new graph. A straight line is the usual result, showing that the period squared is proportional to the mass.

#### Velocity and acceleration

**j** On the 'distance vs. time' graph, the gradient at any point represents the velocity of the oscillating mass. Choose the clearest set of data and use the program to calculate the gradient at every point on the graph.

A plot of the resulting data shows a 'velocity vs. time' graph. Note that the new graph is also sinusoidal. However, compared with the 'distance vs. time' graph, there is a phase difference - the velocity is a maximum when the displacement is zero, and vice versa.

**k** A similar gradient calculation based on the 'velocity vs. time' graph yields an 'acceleration vs. time' graph. Comparing this with the original 'distance vs. time' graph shows a phase difference of  $180^{\circ}$ . This indicates that the acceleration is always opposite in direction to the displacement.

#### **Teaching notes**

**1** This experiment illustrates the value of rapid collection and display of data in assisting thinking about the phenomenon under investigation. Data is collected within a few seconds and the graph is presented simultaneously. Students can observe connections between features on the graph and the actual motion of the mass. For example, the crests and troughs on the graph represent the mass at the extremes of its displacement.

The parameters suggested here usually produce displacements of a few centimetres. The motion sensors can detect these with suitable precision. Small amplitude oscillations produce rather noisy data. Starting with the largest mass, shows the clearest results first.

Software tools for taking readings from the graph are employed: measuring gradients and time intervals. The detail available in the data allows the idea that the periodic time remains constant for a given mass to be tested.

**2** A particularly useful software function is that which calculates the velocity for all points on the graph and plots these as a new graph. A notable feature of the velocity graph is the phase difference from the distance data. This can provoke useful discussion about the change in magnitude and direction of velocity during each cycle of oscillation.

The 'noisiness' of the measurements begins to show more markedly on the velocity graph. The process by which the program calculates the velocity (usually by taking differences between distance readings) should be questioned.

**3** The further derivation of acceleration from the gradients of the velocity graph usually shows even more measurement noise. Nevertheless, the form of the graph convincingly shows the antiphase relation with the distance graph. This is useful for prompting discussion about the conditions for simple harmonic motion (SHM). This can be reinforced by plotting a further graph of acceleration against displacement. The negative gradient straight line supports the basic condition for SHM: acceleration is proportional to displacement, but in the opposite direction.

# **Additional activities**

**4** You could add a card to the bottom of the masses to increase the damping. Students can see if the presence or amount of damping affect the natural frequency. Secondly, the amplitude can be extracted from each peak and a damping curve plotted. This can be tested to see if it is exponential.

This experiment was submitted by Laurence Rogers, Senior Lecturer in Education at Leicester University.

Updated 12 May 2006

http://www.practicalphysics.org/go/Experiment\_220.html?topic\_id=3&collection\_id=52

# Datalogging SHM of a mass on a spring

# **Demonstration or class experiment**

To demonstrate SHM of a mass on a spring and gather accurate data using a datalogger.

# Apparatus and materials

Datalogger position sensor Slotted 50 g masses and hanger Expendable spring Card

# **Technical notes**

Be careful to set the mass moving only vertically, not swinging side to side. Also many position sensors do not work if the object gets too close - be careful to maintain at least the minimum working distance at all times.

# Safety

Masses hanging from ceiling.

# Procedure

**a** Attach an expendable spring to the ceiling or a very high retort stand and hang a 50 g slotted mass hanger from it. Place a motion sensor underneath, pointing upwards. Displace the mass a small distance downwards. Position-time data can be recorded swiftly and easily.

**b** The experiment can be repeated using different numbers of masses, springs in series and adding card to the bottom of the masses to increase the damping.

# **Teaching notes**

If you have motion sensors this is much easier to set up than a pendulum attached to a rotary potentiometer.

This can work at every level. Initially data can be recorded over a few cycles and used to find the period and hence the frequency. The effect of changing the mass, springs and damping is quickly measured and compared directly to theory.

Position-time data can be analysed to see if it is sinusoidal and if the period is constant as the amplitude damps down. Also period being independent of initial amplitude can be checked.

Velocity and acceleration-time graphs can be plotted by the computer and shown to have the same shape and period but different phase to the position-time graph.

The damping can be investigated in various ways. Initially students can look and see if the presence or amount of damping effects the natural frequency. Secondly the amplitude can be extracted from each peak and a damping curve plotted. This can be tested to see if it is exponential.

As the data is on a computer it can be exported to a spreadsheet or maths package and fitted to a sinusoidal form (with exponential damping if appropriate).

This experiment was submitted by Ken Zetie, Head of Physics at St Paul's School in West London. He is on the editorial board of Physics Education and regularly contributes to Physics Review.

Updated 21 Dec 2004 http://www.practicalphysics.org/go/Experiment\_209.html?topic\_id=3&collection\_id=52

# Feeling a force of 10 N

#### Demonstration

This is a fairly elementary demonstration, but it will help students to develop a 'feel' for a newton of force, together with the concept of field strength.

# **Apparatus and materials**

- *Mass*, 1 kg
- *Mass*, 2 kg
- Forcemeter, 0-50 N

#### Procedure



**a** Get a student to hold a kilogram in one hand.

**b** Ask, 'Can you feel the force of the Earth pulling down on it?' and 'How big a force is it?' If the student cannot answer the latter question, attach the mass to a forcemeter.

**c** Prepare a table with 3 columns: label the first column 'mass' and the second column 'gravitational force on the mass'. Leave the third column unlabelled for the moment.

**d** Complete the table for 2, 3, and 4 kg masses.

**e** Now head the third column 'force on a unit mass of 1 kilogram' or 'gravitational field strength'. Ask, 'What is the force on one kilogram of each of the masses?' Complete the third column together.

# **Teaching notes**

**1** Students may have heard the story of Newton 'discovering' gravity when an apple fell on his head as he sat thinking under an apple tree. The weight of an apple is about 1 newton. Newton's remarkable thought about gravity is that the Moon too is falling.

**2** Students might find the question in part **e** puzzling because it is so obvious. They need only imagine the mass cut up into 1-kg lumps. The force on each kilogram is 10 N whether its mass comes in 1-kg lumps or 4-kg lumps. This is a very important

demonstration because it leads to the concept of gravitational field strength. Its symbol is g and its units are N/kg.

Physicists picture the gravitational field of force spreading out from the Earth with a 'readiness to pull' another mass, radially, towards the centre of the Earth. There is no actual force at a place near the Earth until you put some stuff there for the field to pull on. The field is always there.

**3** The force *F* that acts on a mass *m* in a gravitational field is F = mg newtons. This is the weight of the body. When you hold an object and feel the pull of the Earth's gravitational field on it, that object is not falling. It is not accelerating downwards with an acceleration *g* and it is therefore nonsense to talk of finding its weight by multiplying its mass by an *acceleration*, *g*. Here *g* represents the Earth's *field strength*, 9.8 *N/kg*.

Updated 15 Apr 2005

http://www.practicalphysics.org/go/Experiment\_419.html?topic\_id=3&collection\_id=63

# Investigating momentum during collisions

## Demonstration

A moving glider on a linear air track collides with a stationary glider, thus giving it some momentum. This datalogging experiment explores the relationship between the momentum of the initially moving glider, and the momentum of both gliders after the collision.

## **Apparatus and materials**

- *Light gates, interface and computer, 2*
- Linear air track with two gliders, each fitted with a black card

- Glider accessories: magnetic buffers, pin and plasticine
- Clamps for light gates, 2
- Electronic balance

# **Technical notes**

Set up the linear air track in the usual manner, taking care to adjust it to be perfectly horizontal. A stationary glider should not drift in either direction when placed on the track.

Select two air track gliders of equal mass. Attach to each a magnetic buffer at one end, and a black card in the middle.

Prepare each card accurately to a width of 5.0 cm, and enter this value into the software.

The mass of the gliders must also be measured and entered into the software to prepare for the calculations (see below). If magnets are not available, 'crossed' rubber band catapults are an acceptable alternative.

Connect the light gates via an interface to a computer running data-logging software.

The program should be configured to obtain measurements of momentum, derived from the interruptions of the light beams by the cards.

The internal calculation within the program uses the interruption times from each light gate to obtain two velocities. These are multiplied by the appropriate glider masses to give two values of momentum, one before the collision, and one after. This assumes that the measurements for the width of the card and the masses of the gliders have been entered into the program correctly.

For the elastic collision (first part), the momentum measured at A depends upon the mass of the moving glider only. The momentum measured at B depends upon the mass of the initially stationary glider only.

For the inelastic collision (second part), the momentum measured at A depends upon the mass of the moving glider, whereas the momentum measured at B depends upon the combined mass of both gliders.

Students accumulate a series of results in a table with two columns, showing the momentum before and after each collision. It is informative to display successive

measurements on a simple bar chart.



#### Safety

The most significant hazard is that of setting up the linear air track on the bench, especially if it is stored on a high shelf. Two people may be needed to achieve this safely.

### Procedure

#### **Data collection**

#### Part 1 – Elastic Collisions

**a** Position the light gates A and B either side of the midpoint of the track as shown.

**b** Place one glider at the left hand end of the track, and the second between the light gates, with the magnetic buffers facing. The second glider should remain stationary.

**c** Give the first glider a short push so that it passes through light gate A. It then collides with the stationary glider. This then moves and passes through light gate B. If necessary, adjust the positions of the light gates to make sure that the sequence is correct.

(As the magnetic buffers approach each other they repel so that there is no real contact between the two gliders. This creates the condition for 'elastic' collisions.)

**d** Return the gliders to their starting positions, set the software to <u>record data</u>, and repeat the sequence. Observe the measurements of momentum before and after the collision. Repeat this whole process several times to obtain measurements for a series of collisions.

# Part 2 – Inelastic collisions

**e** Replace the magnetic buffers with a pin on one glider and a lump of Plasticine on the other. (This will cause the gliders to stick together after the collision, making it an 'inelastic' collision.) The black card may be removed from the initially stationary glider.

Reset the program so that the measurements at B use the combined mass of both gliders.

**f** Use the same procedure as for Part 1 to obtain measurements for a series of inelastic collisions.

# Analysis

**g** Depending upon the software, the results may be displayed on a bar chart as the experiment proceeds. Note the very similar values for momentum before and after each collision of either type.

**h** The results can be displayed as a graph of 'momentum before collision' against 'momentum after collision'. A straight line graph would demonstrate that the relationship does not depend upon the magnitude of the initial momentum. If the graph is at 45°, this confirms the conservation of momentum.

# **Teaching notes**

**1** This is a computer-assisted version of the classic experiment, using light gates and electronic timers. The great advantage of this version is the instant presentation of momentum values using the software. This avoids preoccupation with the calculation process and allows attention to focus on the results.

**2** It is unusual for the measured values of momentum before and after each collision to be identical. It is wise to limit the number of decimal places displayed, so that the discrepancy does not appear exaggerated. Note how small the discrepancy is, compared with the magnitude of each value. A bar chart display makes this comparison very plain to see. Thus it can be argued that momentum is conserved in each case.

**3** A discussion of the measurement errors must consider the residual friction affecting the motion of the gliders. Errors may be kept to a minimum by strategically placing the light gates so that they capture the motion as close as possible to before and after a collision.

This experiment was submitted by Laurence Rogers, Senior Lecturer in Education at Leicester University.

Updated 12 May 2006

http://www.practicalphysics.org/go/Experiment\_221.html?topic\_id=3&collection\_id=51

# Measuring the density of water 1

# Demonstration

This could be a useful introduction to the idea of density of a liquid.

# Apparatus and materials

- Perspex box with internal dimensions 10 cm x 10 cm x 11 cm
- *Domestic balance (2 kg to +/- 2 g is adequate)*
- Cube, 1 m x 1 m x 1 m (e.g. five squares of Corriflute, 1 m x 1 m, with L-shaped wire links for the vertical edges. The fifth square sits on the top no base is needed.)

# **Technical notes**

The Perspex box should have a horizontal line marked on one face, 10 cm from the internal base so it can be filled to that depth.

The metre cube could be made out of cardboard, and could be collapsible for ease of storage.

# Procedure



**a** Find the masses of the empty Perspex box and then the box filled to a depth of 10 cm.

**b** Place the cardboard cube beside the Perspex box.

## **Teaching notes**

**1** Using cubes will help younger or weaker students make the link with densities found for blocks of solids. They may need reminding that 1000 cubic centimetres = 1 litre.

**2** Having found the mass of 1000 cubic centimetres of water, the metre cube can be used to help students see that the mass of a cubic metre of water would be 1000 times greater. Some may be familiar with the thought that the density of water is  $1 \text{ g/cm}^3$ . This will help them see it cannot be 1 kilogram per cubic metre but rather 1000 kg/m<sup>3</sup> !

**3** You could remind students that building materials like sand and gravel are sold in cubic metres and so have to be transported in strong sacks.

Updated 11 Jul 2006

http://www.practicalphysics.org/go/Experiment\_610.html

# Measuring the density of air 1

#### Demonstration

This simple experiment gives a reasonable value for the density of air.

## **Apparatus and materials**

- 1-litre flask, strong, round bottom
- Vacuum pump, electronically operated rotary type
- Top-pan chemical balance with a sensitivity of no less than 0.01 g
- *Measuring cylinder (1 litre capacity would be useful)*
- Bung, plastic tube and short length of rubber vacuum hose
- *Hoffman clip*
- Water trough
- Safety screens
- Cork ring (see technical notes)

#### **Technical notes**

The bung should fit the mouth of the flask snugly so that it is secure but unlikely to be forced into the flask when that is evacuated.

The glass tube through the bung should be just long enough to allow the rubber tubing to be securely fitted. The rubber tube connecting the glass tube to the pump should be just long enough to give space for the clip to

be used, before detaching the tube from the pump.

A cork ring to support the flask in an upright position will help with the weighings.

Avoid using a flask with a smaller volume. The mass of the air will be correspondingly more awkward to measure, and finding the density is more difficult.

## Safety

Remember that vacuum pumps of this type are heavy. Two persons will probably be needed to move one from trolley to bench.

Safety screens should surround the flask before it is evacuated.

# Procedure

**a** Open the clip (making sure the rubber tube also opens). Weigh the flask, bung, clip, and tubes.

**b** Connect the flask to the vacuum pump to remove as much air as possible from the flask. Tighten the clip and close any inlet valve on the pump. Stop the pump and detach the tubing from the pump inlet.

c Weigh the flask again to find the mass of the air removed.

**d** Invert the flask over water in the trough, so that the rubber tube remains under the water surface. Carefully open the clip to allow water to fill the flask (and tubes). Then empty the water into a measuring cylinder to find its volume.

#### **Teaching notes**

**1** Seeing the water refill the flask is impressive. It also helps those who wonder whether the volume is indeed 1 litre. Even to those for whom it is self-evident that the flask has a volume of about 1 litre

**2** Getting students to realise that the mass of a litre of air is about 1 g, so that the mass of a cubic metre is about 1 kg, is no mean achievement. (Should they find the mass of the litre of air is nearer 1.2 grams that is a pleasant bonus.) Generally, they will be surprised that air is 'so heavy!'

**3** A useful extension is to estimate the volume of the room and hence the mass of the air it contains. This might lead to seeing why, living as we do at the bottom of an ocean of

air, the pressure exerted by the atmosphere is so great. Comparing the density of air with that of water, can lead to a discussion of the hazards of descending into the depths of the sea.

## **4** You could say:

Returning to the calculation of the speed of air molecules, we can move forward another step. Air has a density of about  $1.2 \text{ kg/m}^3$ , and water has a density of about 1 kg/litre. Air is therefore about 830 times less dense than water. The height of a column of water in a water barometer is about 10 m. If we had a uniform atmosphere of air, then it would have to have a height of 10 x 830 m; the height of the atmosphere would be 830 m or 8.3 km.

However, the atmosphere is not of uniform density as can be demonstrated in the 3-D kinetic theory model which you may remember seeing, or we could look at it again. The story we are telling is an artificial, simplified one in order to arrive at an interesting guess. Desperate measures for desperate needs.

Thanks to Neil Johnston for several suggestions that improved this experiment.

Updated 30 Jan 2007

# A quick comparison of densities

#### Demonstration

This shows dramatic differences in the density of air, water and mercury.

# **Apparatus and materials**

- Similar bottles, 3 (about 150 to 200 ml volume)
- Domestic spring balance or a top pan balance (5 kg to +/- 2 g)
- Mercury
- Tray to hold the bottle filled with mercury

# **Technical notes**

The bottles should be filled with air, water and mercury. All should be sealed securely and similarly. (Those sold as 'medical flats' are suitable.)

# Safety

Take the usual care when using mercury, but, as it is in a sealed container, it presents minimal risk.

# Procedure



**a** Find and record the masses of the three bottles.

**b** Get the students to work out how much heavier the mercury is than the water, as a ratio.

# **Teaching notes**

**1** Before the weighings, discuss the purpose of the bottle filled with air. It leads to the assumption that the mass of air is negligible compared with the mass of the container. Hence the mass of water and mercury can be calculated.

**2** If possible, allow students to lift the bottle of mercury. (Insist that the bottle stays with its tray.) Their surprise at how heavy it feels will be memorable.

**3** The volumes of mercury and water are the same, so the ratio of their masses gives the ratio of their densities. The absolute density can be found if their volumes are measured.

# 4 You could say:

Returning to the calculation of the velocity of air molecules at the base of the atmosphere, we now know that the density of mercury is about 13.5 times as dense as water. So, a water atmosphere would have to be 13.5 times as high as the mercury height, 13.5 x 75 cm; about 10 m.

What about an atmosphere of air? Not air that gets thinner and thinner all the way up, but air that stays just as dense as it is here in this room?

Now we are going to need a comparison of the density of water and the density of air. See the experiment <u>Measuring the density of air 1</u>.

Updated 11 Jul 2006

http://www.practicalphysics.org/go/Experiment\_609.html

# **Newton's Laws of Motion**

Dr. Larry Bortner

edited

# Purpose

To demonstrate and verify Newton's laws of motion.

# Background

Sir Isaac Newton was the first to present us with an objective means of predicting the motion of an object under the influence of some force.

- His first law tells us that any change in straight-line motion of an object requires a force.
- The second law quantifies the force and the change in motion and is summarized in the equation

# $\vec{F} = \vec{ma}$ (1)

- where F is the *resultant* (net) force acting on an object of mass m and **a** is the acceleration that results.
- The third law states that for two bodies interacting with each other but nothing else, any force that body number one exerts on the body number two must be accompanied by an equal force from body number two exerted on body number one in the opposite direction.

There are a couple of items of information embedded in Eq. 1 that you should be aware of, which is also mentioned in the Vector Forces experiment:

- If there is no resultant force, the object will not accelerate (i.e., will either remain at rest or in uniform linear motion).
- It is a vector equation, indicating that F and  $\vec{\mathbf{a}}$  are in the same direction.

In this experiment we use Eq. 1 to predict the motion of two different masses, each under the influence of two different forces at separate times. We then verify the predictions using an air track.

An air track permits cars or gliders of various masses to move linearly along its length, and it offers very little frictional resistance to motion. This simplifies the determination of the resultant force on the car, which is the externally applied force minus frictional force. The air track is also equipped with a low friction air pulley.

# Theory, First Force: Gravitation on an Inclined Plane

One way of applying an external force to a car on the air track is to raise one end of the track by putting an aluminum block or *shim* under the support at one end. This creates an inclined plane. The forces in this system are shown in Fig. 1, with an exaggerated angle.



Figure 1 Force diagram of inclined plane.

If the track of length L between the legs is raised at one end by a height **h**, then the angle of inclination is given by:

$$\sin \Theta = \frac{h}{L}$$
. (2)

Since the frictional force is negligible, the only force acting on the car is the gravitational force. The component of this force parallel to the track is given by:

 $\mathbf{F} = \mathbf{mg} \, \sin \, \theta = \frac{\mathbf{mgh}}{\mathbf{L}} \, , \, {}_{(3)}$ 

where **m** is the mass of the car and  $g=9.80 \text{ m/s}^2$  is the acceleration due to gravity. Applying Eq. 1 to this system, we find:

or the theoretical acceleration down the inclined plane is

$$\mathbf{a} = \frac{\mathbf{gh}}{\mathbf{L}} \, . \quad (5)$$

Note that the mass of the car does not appear in Eq. 5, indicating that the acceleration of an object on an frictionless inclined plane is independent of its mass. This is because the force exerted on the object (Eq. 3) is proportional to the mass of the object and the mass cancels from both sides of the equation in Eq. 4.



# Theory, Second Force: Tension from a Falling Mass

Figure 2 Accelerating the car horizontally with a falling mass.

Another method of applying an external force is by adding a known mass  $m_{drop}$  to the free end of a string that is attached to the car and runs over a pulley at one end of the track. This situation is pictured in Fig. 2. Assuming that the string is massless and that the friction over the pulley is negligible, the tension T on the string is the same throughout its length. (If it were not, consider what would happen to a section of the string somewhere near the middle. Address this scenario in your *Conclusions*.) T is the resultant force on m and pulls the glider along the track toward the pulley. Consequently, Eq. 1 applied to m is:

# T = ma . (6)

Of course, the tension also acts on the dropping mass  $m_{drop}$ , its effect being to hold it up. In fact, the string's tension clearly holds the two masses together, pulling each toward the other so that they go no further apart. The second force acting on  $m_{drop}$  is that due to gravity (= $m_{drop}g$ ), pulling down. So the resultant force on  $m_{drop}$  is  $m_{drop}g - T$ *downwards*. Since the string does not stretch,  $m_{drop}$  accelerates *downwards* with acceleration **a**. Therefore, Eq. 1 applied to  $m_{drop}$  is:

$$\mathbf{m}_{drop}\mathbf{g} - \mathbf{T} = \mathbf{m}_{drop}\mathbf{a}_{(7)}$$

Eliminating  $\mathbf{T}$  between Eq's. 6 and 7, we can calculate the acceleration of the car:

$$\mathbf{a} = \frac{\mathbf{m}_{drop} \mathbf{g}}{\mathbf{m}_{drop} + \mathbf{m}} \ . \tag{8}$$

# **Measuring Acceleration**

Looking at the theoretical predictions of Eqs. 5 and 8, we need to experimentally measure the acceleration of a car traveling along the air track in order to verify Newton's Laws. The method we're familiar with from the **Falling Bodies** experiment is to measure the time it takes for the car to travel various distances and plot velocity vs. time, with the calculated slope being the acceleration. However, in the spirit of investigation and expanding our horizons, we'll do it a different way.

We use a different equation of motion for an object under a constant acceleration than we used previously. (It is important to note that these equations are independent of Newton's Laws.) If the car travels a distance  ${\bf x}$  starting at rest, the square of the theoretical speed  ${\bf v}$  at this point is

# $v^2 = 2ax$ (theoretical velocity). (9)

A plot of  $v^2$  vs. 2x then gives the experimental acceleration.

To find the experimental velocity, attach a flag of known length flag to the car. This flag interrupts the beam in a photogate when the car passes through it. The instantaneous velocity **v** of the car at the detector when the flag is halfway through is the average velocity over the total time **†** that the beam was broken,

# v = flag/t (experimental velocity). (10)

The direction of both types of forces we're investigating are going to the right as you're facing the air track. A photogate is set up at the right end of the track. To determine the distance traveled  $\times$ , we need the initial and final positions of the car. The initial position is simple enough. Where the car breaks the photogate beam determines the final position.

Define  $x_1$  as the position of the car as it goes through the photogate from left to right, when the flag first interrupts the photobeam (the timer *starts* timing). Define  $x_2$  as the position of the car as it goes through the photogate from left to right, when the flag first stops interrupting the photobeam (the timer *stops* timing).

The *position* of the car when it has the velocity **v** in Eq. 10 is the average of the positions when it enters the photogate  $(x_1)$  when it leaves  $(x_2)$ .

Measure the position of the glider where its left edge aligns with the metric ruler that is set in the air track. Note that the positions  $x_1$  and  $x_2$  are different for each car but are the same for both forces acting on a single car.

Assume that a car starts at zero velocity at the initial position  $x_0$  (measured from the left edge!) and accelerates through the

photogate. Double the distance traveled by the glider to where the flag is halfway through the photogate is

 $2\mathbf{x} = 2\left[\left(\mathbf{x}_1 + \mathbf{x}_2\right)/2 - \mathbf{x}_0\right]$  $= \mathbf{x}_{2} + \mathbf{x}_{1} - 2\mathbf{x}_{0}$ .

The error in this calculation is

 $u \{2x\} \approx 2.5 \cdot u \{x\}$ . (12)

# Procedure

You need the following equipment:

- ► air track
- ▶ photogate
- digital calipers
- metric tape measure
- 2 cars (gliders) of unequal mass with flags (aluminum inserts on top)
- ► 3 binder clips
- piece of string with tied loops at both ends
- various aluminum blocks
- ► triple beam balance
- Computer with Science WorkShop interface

*Note: never mark the surface of the air track! Even the thinnest layer of ink or graphite significantly adds to the friction. Scratching the surface disrupts the air flow, which can slow things down.* 

# **Basic Measurements**

- 1. The photogate should be just to the right of the crossbar at the right end of the air track.
  - a. For each car, measure the positions x1 and x2 in meters (not millimeters, not centimeters), always with the car side marked with an X facing out. Again, this is where the car first causes the timer to start, then where the timing first stops after you slide the car through the photogate.
  - b. The numbering on the rule on the track can be confusing. There are little 1m's and 2m's at strategic locations meaning

that the position is one or two meters *plus* the millimeters or centimeters that you are reading.

- c. Do not move the photogate during the experiment. The value of  $x_1$  for each car should be the same for the second force as for the first force.
- 2. Measure and record the masses of your two cars, with flags attached and with a clip on the flag.
- 3. Press the mm/in button to turn on the digital calipers display.
  - a. With the caliper jaws closed, press the **0** button to get a zero display.
- Choose a number (1-5) of aluminum blocks to place under the single foot at the left end of the track. This raises that end a distance h. Measure h ( the thickness of the blocks stacked together) with the calipers and record this number.
- 5. Using the tape measure, measure and record the distance L between supports of the air track (where it touches the table; refer to Fig. 1). Record your estimated uncertainty.
- 6. Turn on the air and make sure the track is level. A glider will drift a little back and forth on a level track. Consult your TA if the glider accelerates to one end. *Do not attempt to adjust the air track yourself.*
- 7. Place the blocks under the left end support of the track.
- 8. Click on Start> Science Workshop> First Quarter> Newton's Laws timer to start the DataStudio program.
  - a. Click on the **Start** button to start the timer.
  - b. Leave it running for the rest of the experiment. It automatically resets, displaying the last time that the photobeam was interrupted.

# **Experiment, First Force: Gravitation on an Inclined Plane**

- Position the left edge of the small car at the 2.150 m mark. This gives you a distance of a few centimeters to the photosensor. This is your first initial position (release point) x<sub>0</sub>.
  - a. Record this position in meters. (The air track is over 2½ meters long. As noted previously, the whole number of meters on the rule is not as prominent as the centimeter numbers. Don't miss this number.)
  - b. Release the car. Be careful not to impart any sideways movement or bounce to the car, as this would cause the car to touch the track and introduce friction.
- 10. Stop the car before it goes back through the photogate. Record the displayed time.

- Repeat the timing procedure in Steps 9 and 10 nine more times for the same starting position x<sub>0</sub>. The standard deviation u{t} of the ten times for this distance is the estimated absolute error for all subsequent measured times in this experiment.
- 12. Make single time measurements for seven more different release points **spread out over the length of the air track**. One of these release points should be at the extreme left of the track where the car bumper is still not touching anything. The points should be about 30 centimeters apart. This gives you eight total release/time points.
- 13. Switch to the other car and choose eight different release points. Record these positions and measure and record the times (single measurements).

# **Experiment, Second Force: Tension from a Falling Mass**

- 14. Turn off the blower and remove the aluminum blocks from under the air track.
- 15. Place one of the cars at the end of the track near the pulley.
- 16. Measure the mass of the binder clip  $m_{drop}$  that will be used to accelerate the cars. Remove the string first if it is attached.
- 17. Open the clip and clip it on (through) one of the string loops.
- 18. Remove the clip that is on the glider flag and clip it on the other loop of the string. Reattach the clip to the flag and run the string over the pulley so the accelerating mass is hanging freely.
- 19. Choose eight different release points spread out evenly over the extent of movement (should be about 12.5 cm apart). Measure the time it takes to accelerate through the photogate from each of these points.
- 20. Repeat this for the other car, using the same release points.

# Analysis

1. Click on Start> Templates> First Quarter> Newton's Laws to load the Excel template.

# Data Entry

- 2. Enter in the following:
  - a. the values of  ${\color{black}{h}}$  and  ${\color{black}{L}}$  and their uncertainties
  - b. the uncertainty u{x} in a car position,
  - c. the beam-breaking positions  $x_1$  and  $x_2$  for both cars.
- 3. For the small car, enter in the starting position and the ten times you measured in Steps 9-11 of the **Procedure**.

- 4. Enter in the eight pairs of position and time points for both cars for the first force.
- 5. For the second force, enter in the masses and the eight pairs of position and time points for both cars.

# **Calculations and Comparisons**

- 6. Convert data in millimeters to meters.
- 7. Calculate the theoretical acceleration on the inclined plane from Eq. 5.
  - a. Calculate the error of this value from

$$u \{a\} = a \sqrt{\left(\frac{u \{h\}}{h}\right)^2 + \left(\frac{u \{L\}}{L}\right)^2}$$
.

- 8. Determine the uncertainty u{2x} from Eq. 12.
- Calculate the average time and the standard deviation u(+) for the ten times of the small car at the same release point.
- 10. Calculate the flag length and its uncertainty for both cars.
- 11. Put formulas for 2x (Eq. 11), the experimental speed squared (v<sup>2</sup> using Eq. 10, not Eq. 9), u(v<sup>2</sup>), v<sup>2</sup>+u(v<sup>2</sup>), and v<sup>2</sup>-u(v<sup>2</sup>) in H4, I4, J4, K4, and L4. Be sure to use absolute cell references where appropriate because these formulas are going to be copied. The expression for u(v<sup>2</sup>) is

$$u\left\{v^{2}\right\} = 2v^{2}\sqrt{\left(\frac{u\left\{flag\right\}}{flag}\right)^{2} + \left(\frac{u\left\{t\right\}}{t}\right)^{2}}$$

- a. Highlight these five cells and copy them.
- b. Click in H19 and paste. All five formulas should be copied to the first row of the big car data and calculations done for the first starting position and its time. The formulas have to be corrected but we'll do that in Step 10e.
- c. Click in H52 and paste. Again, three numbers appear, but these are the correct calculations.
- d. Before clicking anywhere else, click on the lower right corner of the three cell selection, drag down to J59, and release. You've just done the calculations for the small car for the second force.
- e. Return to H19 and correct the formulas in this row as needed.
- f. Select the five cells with formulas in that row and copy them.

- g. Go to H66 and paste.
- h. Copy these cells down to row 73. If you scroll down, you'll find that the  $v^2$  vs. 2x data has been plotted for the second force. You should have two different lines.
- i. Copy the cells down for the small car and the big car for the first force.
- 12. Although the plots are automatic, you still have to add the error bars. The X Error Bar of u{2x} is a single Fixed value: for both plots. The u{v<sup>2</sup>} values have to be entered as Custom (+ and -).
- 13. These graphs should be straight lines, since  $v^2=2ax$ . The slopes of these lines are the corresponding accelerations.
  - a. Find the least squares slopes  $a_{exp}$ + and  $a_{exp}$  of  $v^2+u\{v^2\}$  vs. 2x and  $v^2-u\{v^2\}$  vs. 2x. You need just the slope, nothing else. This is one of the few times where you can use LINEST without adding the 1 and 1 at the end and hitting Shift/Control/Enter.
  - b. Find the average of these two slopes for the experimental value of **a**.
  - c. The standard error of these two values is  $u\{a_{exp}\}$ .
- 14. Copy the four formulas. Paste into cells G29, G62, and G76. You have just finished the similar calculations for the other three data sets.
- 15. Are the accelerations of the two cars due to the first force equal to within the accuracy of the experiment?
- 16. Compare both of these accelerations separately to the theoretical value of an object on an inclined plane.
- 17. For the second force, calculate the theoretical acceleration for both cars from Eq. 8 and compare to your experimental values from the least squares fit. Use the following expression for the uncertainty of the theoretical acceleration due to the second force:

$$u \{a\} \sim a \frac{u \{m\}}{m_{dros}}$$

# Questions

- 1. If the string used to connect the small weight to the car in Part 2 had appreciable mass, would the car still undergo uniform acceleration? Explain.
- 2. The experimental value of the acceleration depends differently on the mass of the car for the first force than it does for the second force. Explain how this difference comes about.

http://www.physics.uc.edu/~bortner/labs/Physics%201%20experime nts/Newton's%20Laws%20of%20Motion/Newton's%20Laws%20o f%20Motion%20htm.htm