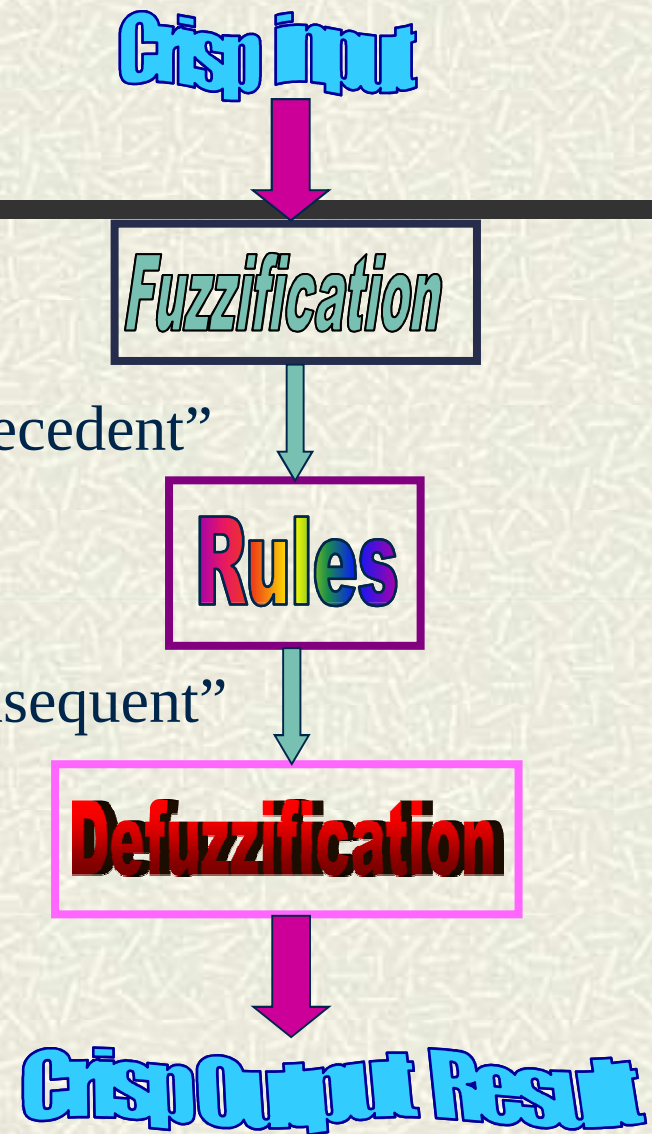


Fuzzy Logic

E. Fuzzy Inference Engine



Fuzzy Inference



Fuzzy Inference Example

- Assume that we need to evaluate student applicants based on their GPA and GRE scores.
 - For simplicity, let us have three categories for each score [High (H), Medium (M), and Low(L)]
 - Let us assume that the decision should be Excellent (E), Very Good (VG), Good (G), Fair (F) or Poor (P)
 - An expert will associate the decisions to the GPA and GRE score. They are then Tabulated.
-

Fuzzy Inference Example

Fuzzy if-then Rules

If the GRE is HIGH and the GPA is HIGH then the student will be EXCELLENT.

If the GRE is LOW and the GPA is HIGH then the student will be FAIR.

etc

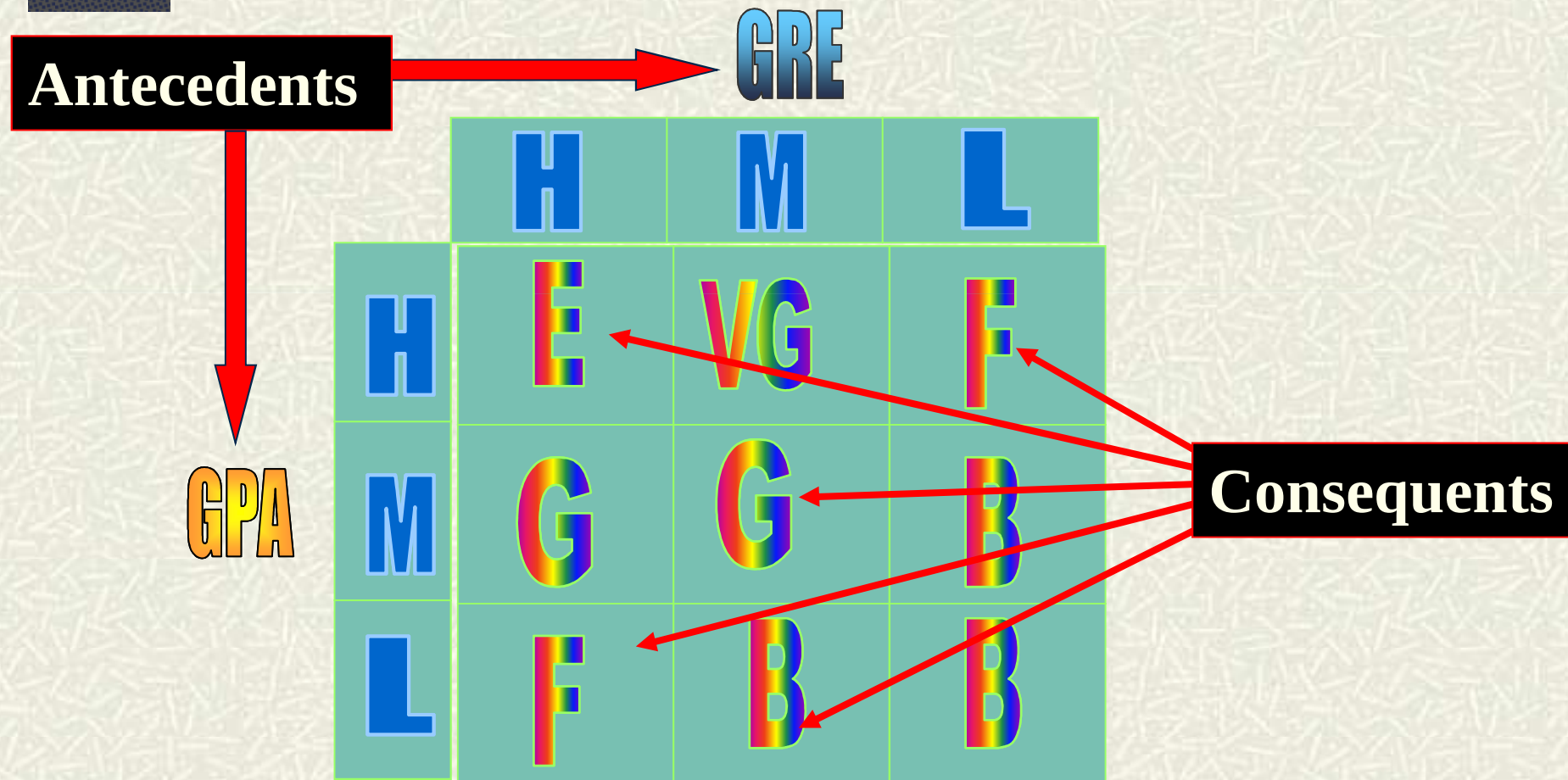
Fuzzy Linguistic Variables

Fuzzy Logic

Antecedent

Consequent

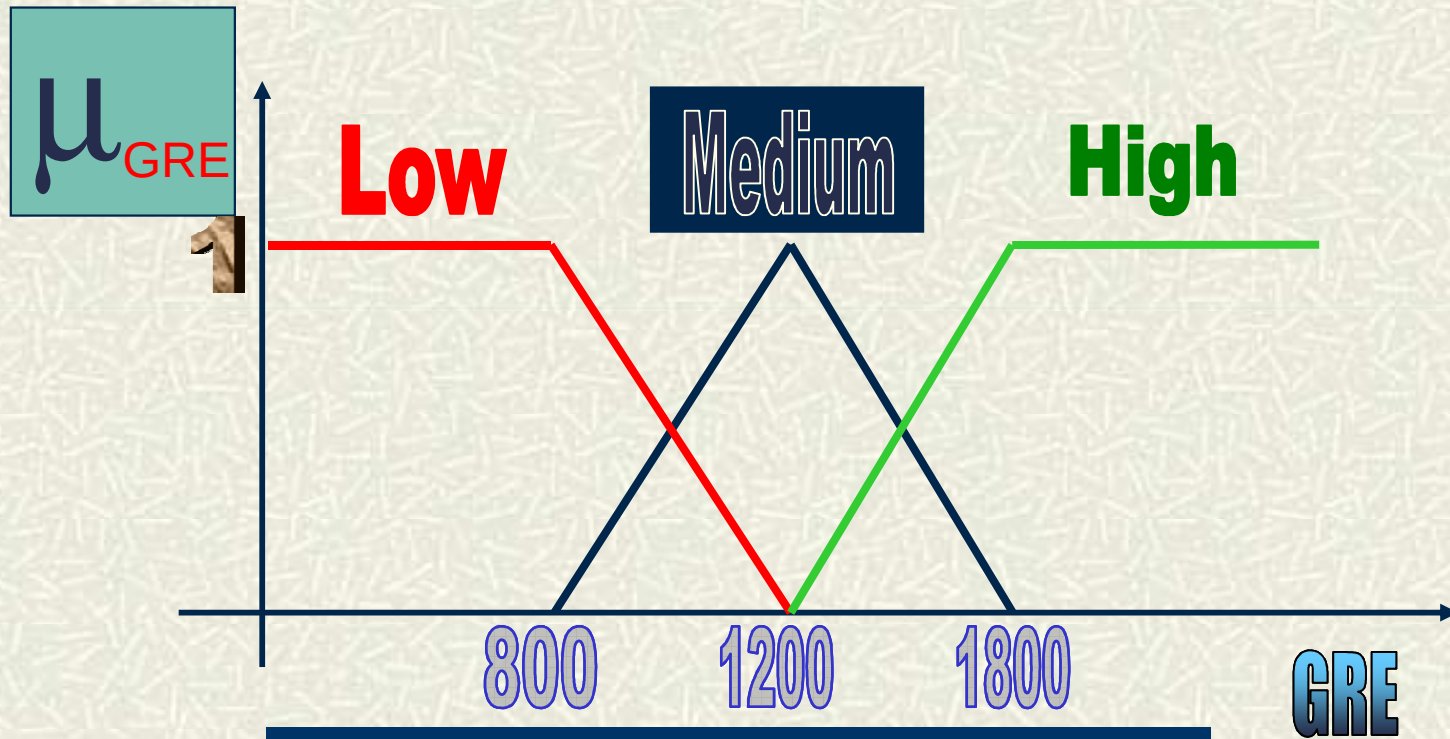
Fuzzy Rule Table



Fuzzification

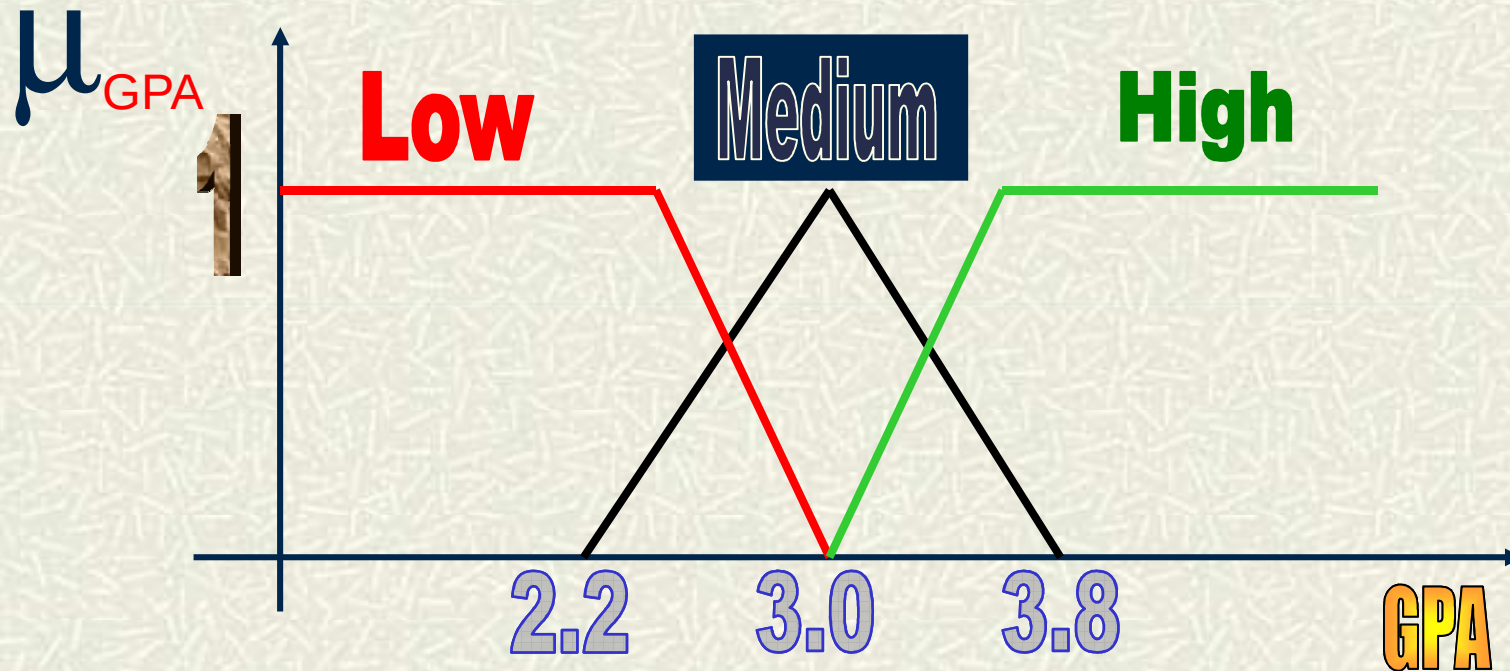
- # Fuzzifier converts a crisp input into a vector of fuzzy membership values.
 - # The membership functions
 - reflects the designer's knowledge
 - provides smooth transition between fuzzy sets
 - are simple to calculate
 - # Typical shapes of the membership function are Gaussian, trapezoidal and triangular.
-

Membership Functions for GRE



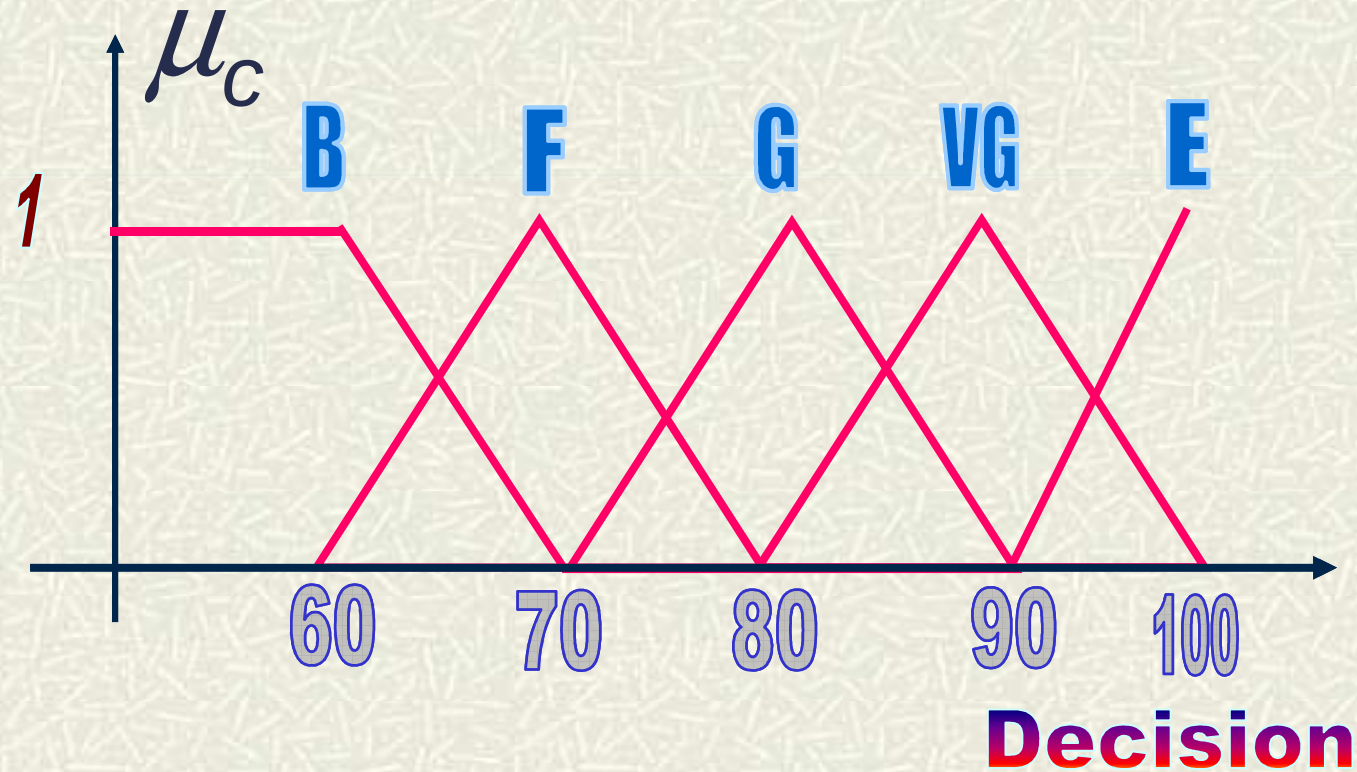
$$\mu_{GRE} = \{\mu_L, \mu_M, \mu_H\}$$

Membership Functions for the GPA



$$\mu_{\text{GPA}} = \{\mu_L, \mu_M, \mu_H\}$$

Membership Function for the Consequent



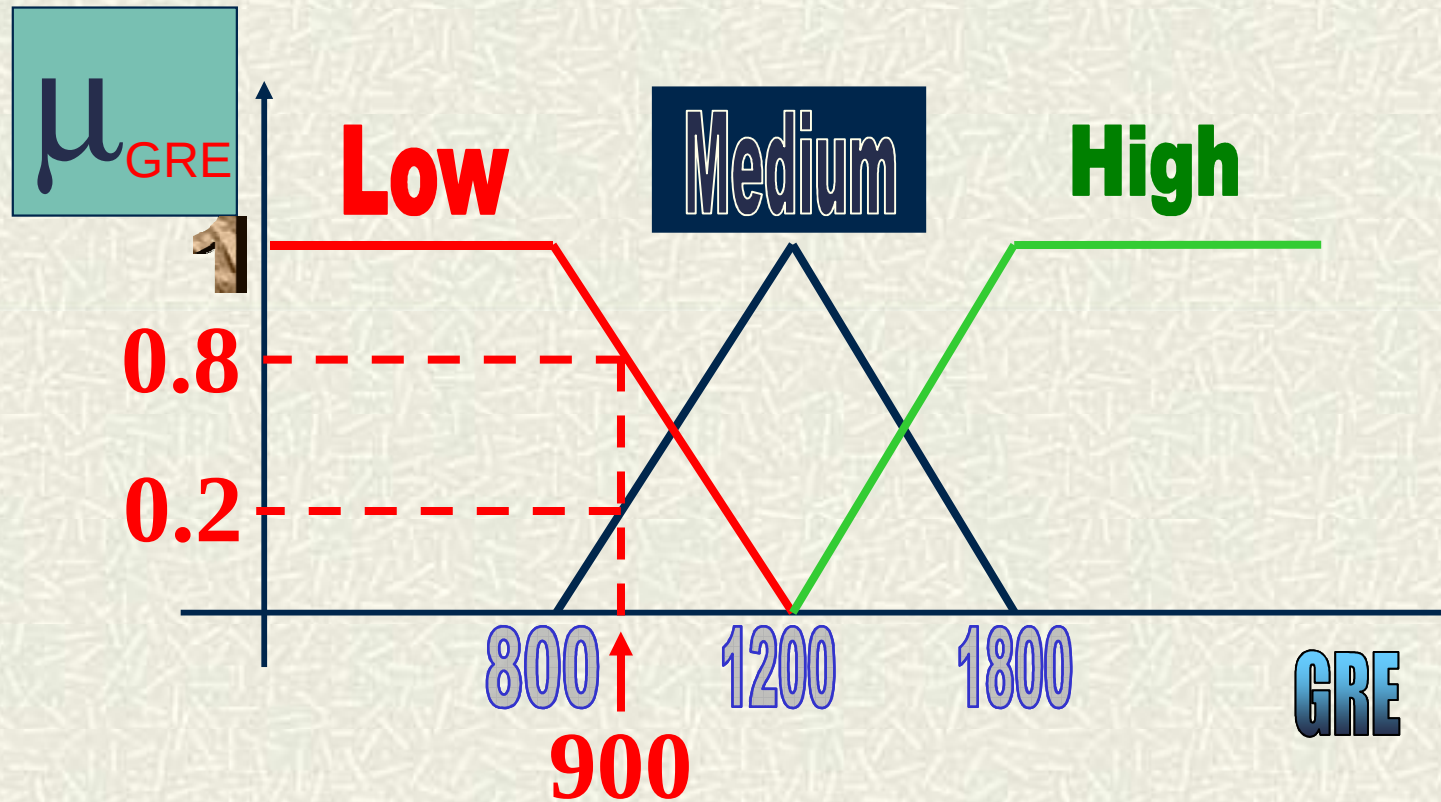
Fuzzification

- # *Transform the crisp antecedents into a vector of fuzzy membership values.*
- # *Assume a student with GRE=900 and GPA=3.6. Examining the membership function gives*

$$\mu_{\text{GRE}} = \{\mu_{\text{L}} = 0.8, \mu_{\text{M}} = 0.2, \mu_{\text{H}} = 0\}$$

$$\mu_{\text{GPA}} = \{\mu_{\text{L}} = 0, \mu_{\text{M}} = 0.6, \mu_{\text{H}} = 0.4\}$$

Fuzzification



Fuzzification

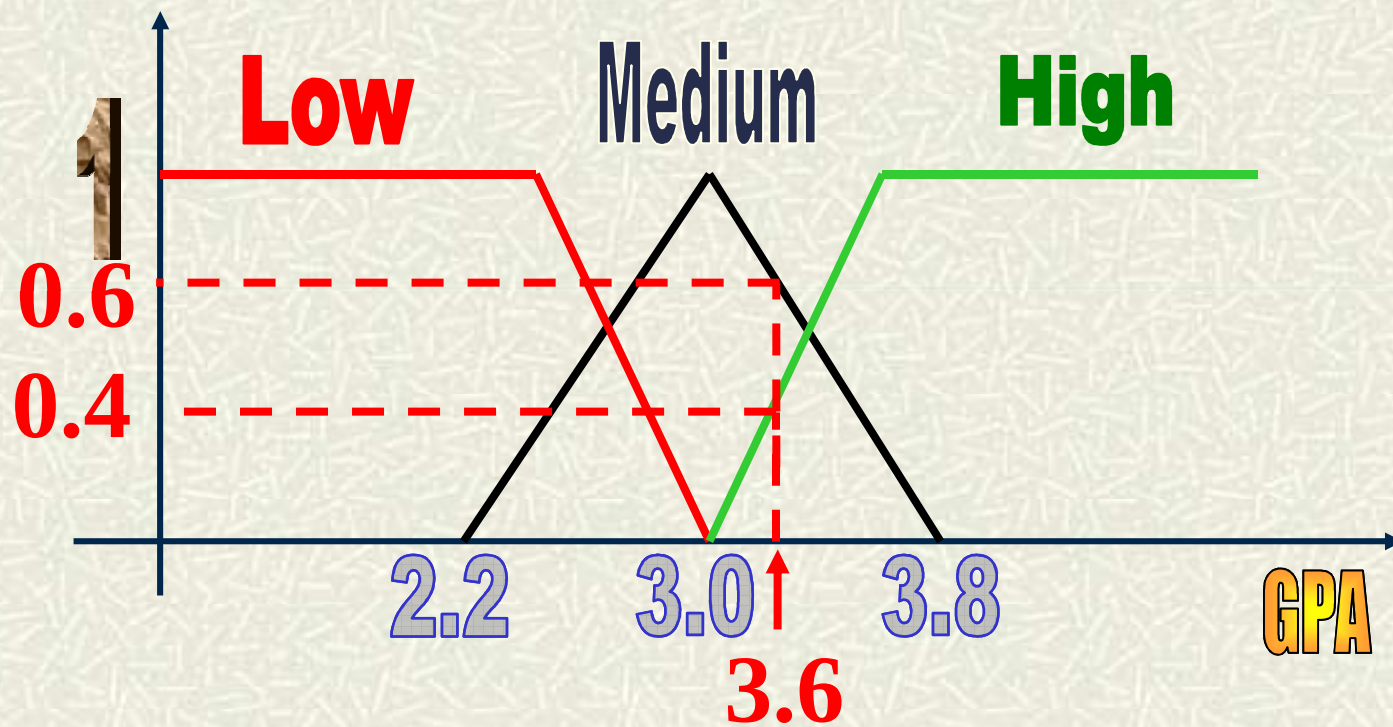


Table:

		GRE			
		0.8	0.2	0.0	
GPA		H	M	L	
	0.0	H	E	VG	F
	0.6	M	G	G	B
	0.4	L	F	B	B

Table:

		GRE		
		0.8	0.2	0.0
		H	M	L
GPA	0.0	H E 0.0	VG 0.0	F 0.0
	0.6	M G 0.6	G 0.2	B 0.0
	0.4	L F 0.4	B 0.2	B 0.0

Rule Evaluation

The student is
GOOD if

(the GRE is **HIGH**
and the GPA is
MEDIUM)

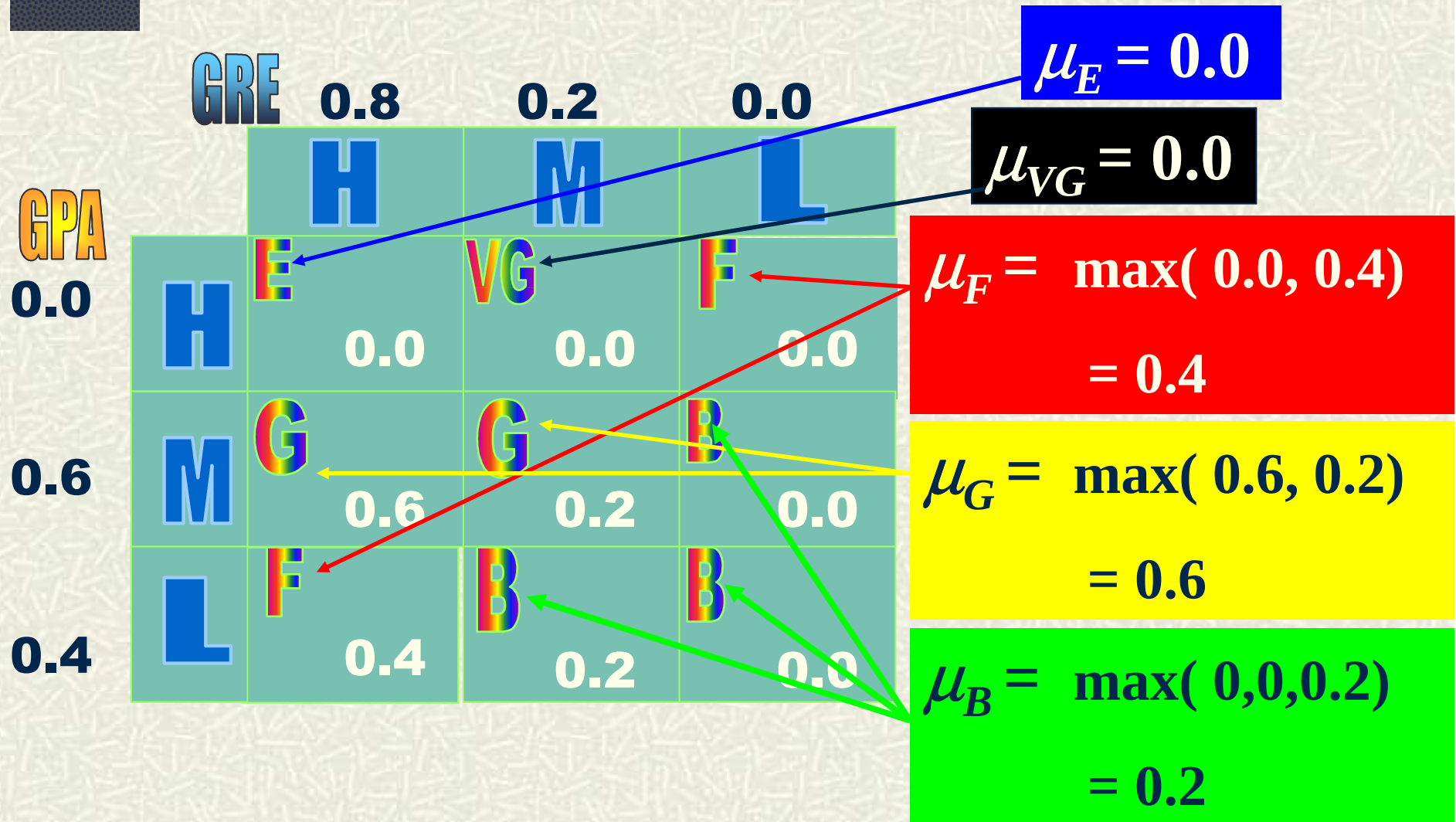
OR

(the GRE is
MEDIUM and the
GPA is **MEDIUM**)

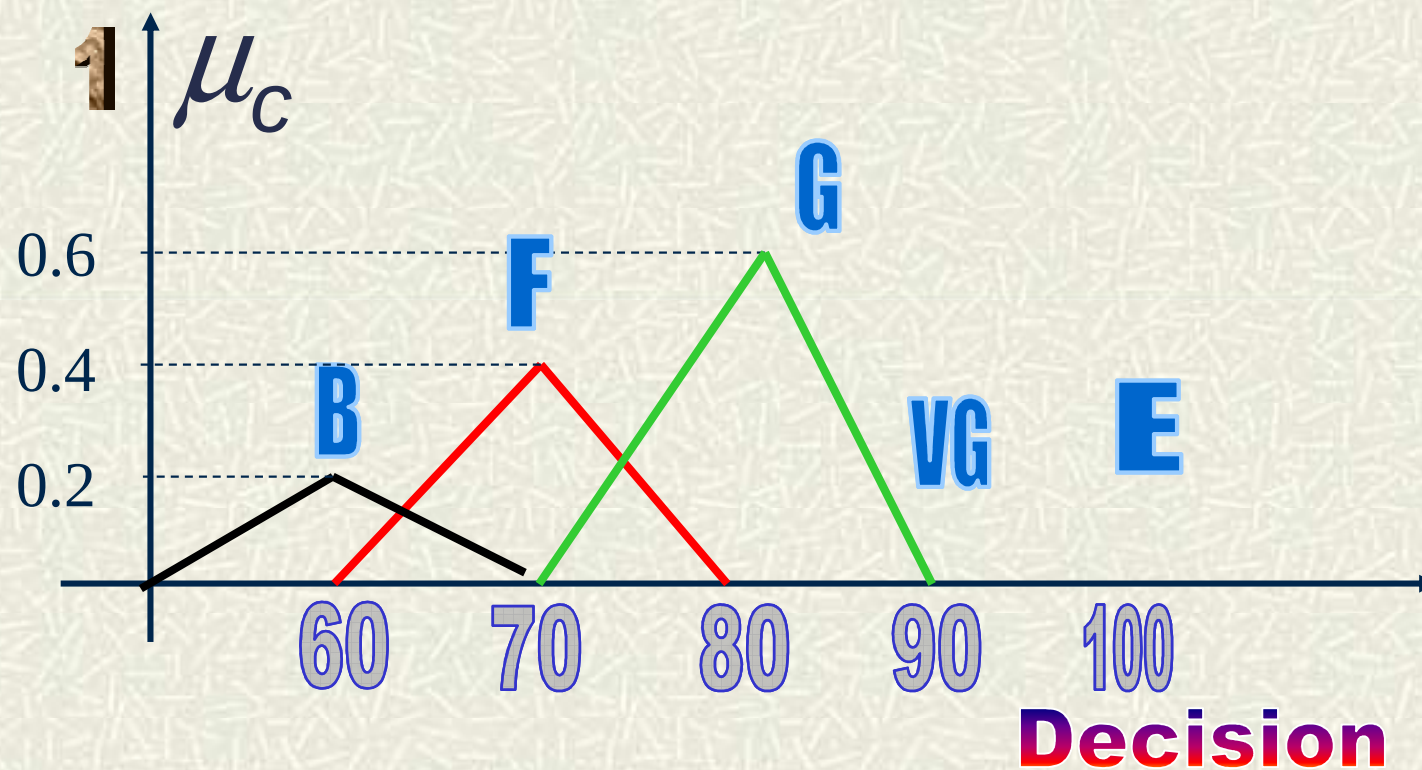
The consequent
GOOD has a
membership of
 $\max(0.6, 0.2) = 0.6$

GRE		0.8	0.2	0.0
		H	M	L
GPA	0.0	H E 0.0	VG 0.0	F 0.0
	0.6	M G 0.6	G 0.2	B 0.0
	0.4	L F 0.4	B 0.2	B 0.0

Rule Evaluation

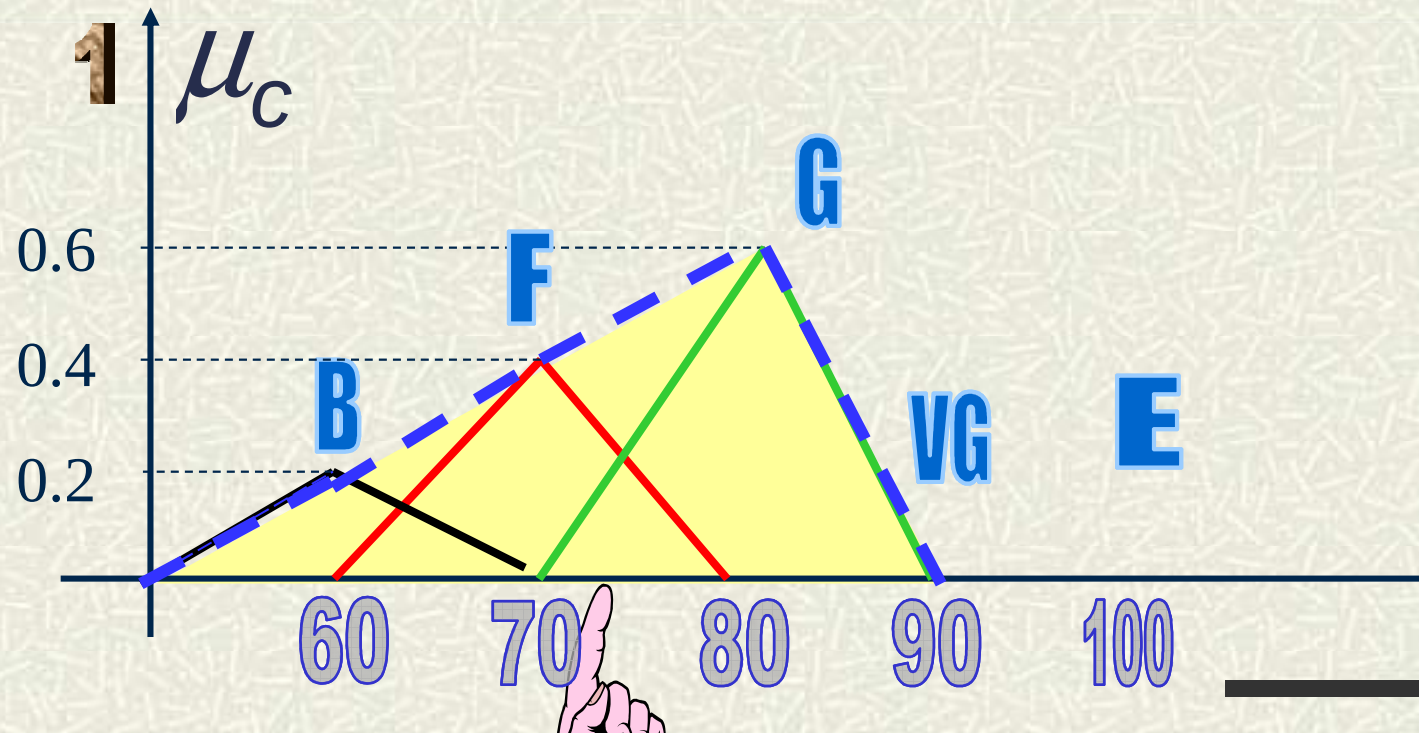


Weight Consequent Memberships



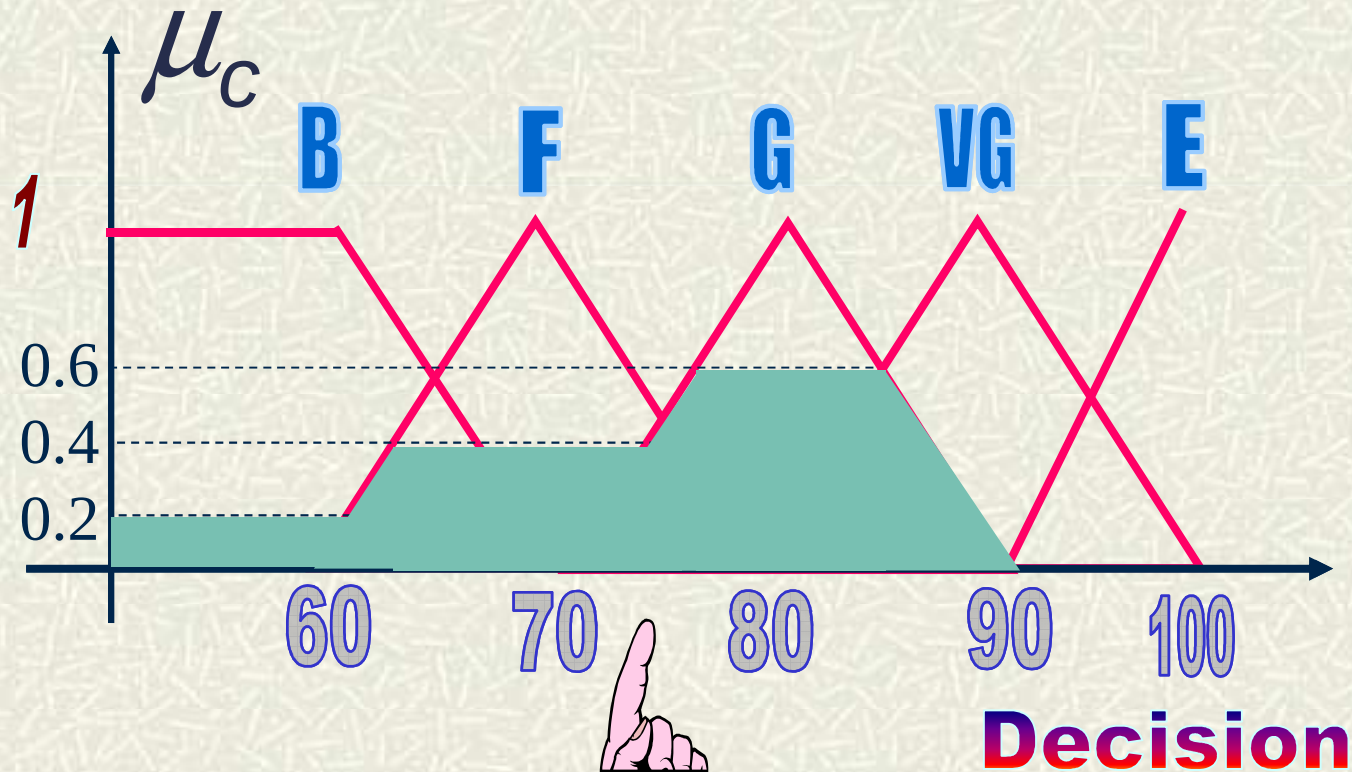
Defuzzification

- ✦ Converts the output fuzzy numbers into a unique (crisp) number
- ✦ **Center of Mass Method:** Add all **weighted** curves and find the center of mass



Defuzzification

Center of Mass Method: Add all *clipped* curves and find the center of mass



Center of maxima and Mean of Maxima

- # An Alternate Approach: Fuzzy set with the largest membership value is selected.
 - # Fuzzy decision:
 $\{B, F, G, VG, E\} = \{0.2, 0.4, 0.6, 0.0, 0.0\}$
 - # Final Decision (FD) = Good Student
 - # If two decisions have same membership max, use the average of the two.
-

Example: Fuzzy Table for Control

		CE							
		LN	MN	SN	ZE	SP	MP	LP	
E	LN	LN	LN	LN	LN	MN	SN	SN	
	MN	LN	LN	LN	MN	SN	ZE	ZE	
	SN	LN	LN	MN	SN	ZE	ZE	SP	
	ZE	LN	MN	SN	ZE	SP	MP	LP	
	SP	SN	ZE	ZE	SP	MP	LP	LP	
	MP	ZE	ZE	SP	MP	LP	LP	LP	
	LP	SP	SP	MP	LP	LP	LP	LP	

Membership Functions

